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PREFƋS.

Ɨ hav loŋ bin sɛkiŋ ɑ ‘Jɛomɛtri fər beginerz, sɥted tɯ mį tɛst, and tɯ mį konvikʃonz ov hwot iž ɑ proper fɵndaʃon fər sįentifik edyɥkaʃon. Fįndiŋ đat Mr. Jɷsįɑ Hɷlbrɯk agrɛd mɷst kərdyali wiđ mɛ in mį estimɛt ov đis studi, į had hɷpt đat hiz trɛtiz wɯd satisfį mɛ; but əlđɷ đɛ best į had sɛn, it did not mɛt mį vɥz. Mɛnhwįl, mį ɷn çildren wẹr in mɷst urjent nɛd ov ɑ tekst bɯk, and đɛ sens ov đɋr wont haz drivn mɛ tɯ tak đɛ tįm nesesari fər rįtiŋ đɛz pajez. Tɷ çildren, wun ov fįv, đɛ uđer ov sevn and ɑ hɋf, wẹr befɷr mį mįndz į əl đɛ tįm ov mį rįtiŋ; and it wil bɛ fɵnd đat çildren ov đis aj ɋr kwiker in komprɛhendiŋ fẹrst lesonz in Jɛomɛtri đan đɷz ov fiftɛn. Meni pɋrts ov đis bɯk wil, hɵever, bɛ fɵnd adapted, not ɷnli tɯ çildren, but tɯ pɥpilz ov adult aj. Đɛ trɷđz ɋr sublįm. Ɨ hav trįd tɯ prɛzent đem in ɑ simpl and atraktiv dres.

Ɨ hav adrest đɛ çįldz imajinaʃon, rɑđer đan hiz rɛzn, bekɛz į wiʃt tɯ tɛç him tɯ konsɛv ov fərmz.

Ƌe çildz pʊerz ov sensaʃon ɋr developt befɷr hiz pʊerz ov konsepʃon, and ðez befɷr hiz reznįŋ pʊerz. Ƌis iz, ðɋrfɷr, ðe trɷ erder ov edyɥkaʃon; and ɑ pʊerfɯl lojikal dril, lįk Kɷlburnz admirabl fęrst lesonz in Ariϑmetik, iz sadli ʊt ov plas in ðe handz ov ɑ çild, hɷz pʊerz ov obzervaʃon and konsepʃon hav, az yet, resevd nɷ tranįŋ hwotever. Ɨ hav, ðɋrfɷr, avɵded reznįŋ, and simpli givn interestįŋ jeɷmetrikal fakts, fited, į hɷp, tɯ arʊz ɑ çild tɯ ðe obzervaʃon ov fenomenɑ, and tɯ ðe persepʃon ov fermz az real entitiz.

ƋƐ ΘĦOR.

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ЄAPTER I.

HWOT ƷIS BƜK IZ ABȢT.

1. Ɨ hav ritn ɑ litl bɯk fer yꝏ abȣt Jeometri. Yꝏ wil fịnd ɑ̣ grat meni nų wurdz in it; but ị hav takn panz tɯ eksplan đem el, and ị ŧiŋk yꝏ wil understand đem el, if yꝏ wil ωnli begin at đe beginiŋ, and red everi çapter veri kąrfɯli befωr yꝏ gω tɯ anuđer. And if yꝏ fịnd eni plas in đe bɯk đat yꝏ kan not understand, ị ŧiŋk yꝏ wil dꝏ wel tɯ turn bak, and red đe hωl ωver agen, from đe sekond çapter. Hwen yꝏ kum agen tɯ đe plas hwiç yꝏ did not understand befωr, ị ŧiŋk yꝏ wil fịnd it haz grωn ezier fer yꝏ.

2. Ɨ hωp yꝏ wil fịnd đe bɯk interestiŋ. It telz abȣt strat lịnz, and sẹrklz, and meni diferent kurvz, and ɑ fų solid bodiz. It wil tel yꝏ kųrius ŧiŋz abȣt đe ʃadωz ov mąrblz, and đe rωliŋ ov hωps, and abȣt tosiŋ ɑ bel, and uđer plaz fer çildren. But, if sum pąrts ov đe bɯk dꝏ not sem interestiŋ, yꝏ ʃɯd studi đωz pąrts el đe mωr kąrfɯli; fer đa ma, perhaps, be đe mωst yųsfɯl pąrts.

3. Jɛometri iz đɛ mωst yųsful ov el đɛ sįensez. Tɯ understand Jɛometri wil be ɑ grat help in lęrniŋ el đɛ uđer sįensez; and nω uđer sįens kan be lęrnt unles yω nω sumŧiŋ ov Jɛometri. Tɯ studi it, wil mak yωr į kwikėr in seiŋ ŧiŋz, and yωr hand stedier in dωiŋ ŧiŋz. Yω kan dre beter, rįt beter, kut ȣt klωđz, mak bωts and ʃωz, wurk at eni mɛkanikal trad, er lęrn eni ɋrt đɛ beter fer understandiŋ Jɛometri. And, if yω wont tɯ understand abȣt plants and animalz, and đɛ wunderful wa in hwiç đɛ Θl-wįz Krɛator haz mad đem, yω must lęrn ɑ litl Jɛometri, fer đat eksplanz đɛ ʃaps ov el ŧiŋz.

4. In đis bɯk į kan teç yω but litl. Į hωp, hȣever, đat it wil be ɛnuf tɯ mak yω wont tɯ nω mωr. Į ʃal tel yω ωnli đɛ eziest and mωst interestiŋ ŧiŋz nȣ; but hwen yω ɋr ωlder yω ma studi hwot iz mωr difikult. Meni ov đɛ ŧiŋz đat į ʃal tel yω wil be veri kųrius, and yω wil, perhaps, wunder hȣ men kan fįnd ȣt suç ŧiŋz. But hwen yω ɋr ωlder, į hωp đat yω wil be abl tɯ fįnd ȣt suç ŧiŋs yωrselvz.

ÇAPTER II.

PƟNTS, LĮNZ AND PLĄNZ.

1. Ɑ pɵnt iz ɑ plas wiđʊt eni sįz. Hwen į mak ɑ dot tɯ mąrk đɛ plas ov ɑ pɵnt, đat iz, tɯ ʃɷ hwąr đɛ pɵnt iz, yɷ must not ŧiŋk đat đɛ pɵnt iz sɷ big az đɛ dot. Đɛ pɵnt haz nɷ plas at ɵl, but iz ɷnli ɑ plas wiđʊt eni sįz. If į put mį dot in đɛ rįt plas, đɛ pɵnt wil bɛ egzaktli in đɛ midl ov đɛ dot. In komon tɵkiŋ, wɛ sumtįmz kɵl eniŧiŋ đat iz veri smɵl ɑ pɵnt; and sɷ wɛ tɵk ov đɛ pɵnt ov ɑ nɛdl, ɵr ov đɛ pɵnt ov ɑ led pensil. But in Jɛometri ɑ pɵnt iz ɑ plas sɷ smɵl đat it haz nɷ sįz at ɵl; nɛđer widŧ, nɵr leŋŧ, nɵr depŧ.

2. Ɑ lįn iz ɑ plas đat iz loŋ wiđʊt haviŋ eni bredŧ ɵr ŧiknes. Hwen į mak ɑ loŋ, fįn strɷk wiđ ɑ pen ɵr pensil, ɵr wiđ ɑ pes ov çɵk, yɷ must not ŧiŋk đat đɛ strɷk ov pensil, ɵr iŋk, ɵr çɵk-mąrk, iz itself đɛ lįn. Ŧɛ ɷnli mak it tɯ ʃɷ hwąr đɛ lįn iz, ɵr tɯ help yɷ imajin ɑ lįn: but đɛ lįn itself iz đɛ

¿Hɤ ląrj is ɑ pɵnt. ¿Hɤ kan yɷ mąrk đɛ pɷziʃon ov ɑ pɵnt. In ¿ hwot pąrt ov đɛ dot is đɛ pɵnt supɷzd tɯ bɛ. ¿Hwot is đɛ diferens betwɛn đɛ wurd pɵnt in komon tɵk and in Jɛometri. ¿Hɤ wįd is ɑ lįn. ¿Hɤ ʃal wɛ mąrk ɑ lįn. In midl ov đε strɷk ; yɷ kan not sε it eni mɷr đan yɷ kan sε ɑ pɵnt, fer it iz ɷnli ɑ plas ; and, elđɷ it haz leŋŧ, it haz nɷ bredŧ ner ŧiknes.

3. Đε endz ov ɑ lįn qr pɵnts. Yɷ ma fansi ɑ lįn tɯ be đε midl ov ɑ veri fįn wįr, and đen yɷ wil ezili sε đat đε endz ov it qr pɵnts.

4. Hwen đε pɵnt ov mį pen er pensil mɷvz aloŋ on paper, ɑ fįn strɷk ov iŋk er ov pensil mqrk iz left behįnd it. And đat ma help yɷ fansi ɑ real pɵnt mɷviŋ aloŋ and leviŋ ɑ real lįn behįnd it. It wil, yɷ nɷ, be ɷnli fansi; bekez ɑ real pɵnt iz ɷnli ɑ plas, and ɑ plas kan not mɷv. But it iz ɑ gɯd wa tɯ fansi ɑ lįn az mqrkt ɤt bį đε trak ov ɑ mɷviŋ pɵnt; đat iz, bį đε veri senter ov đε end ov ɑ pensil. It wil help yɷ veri muç in understandiŋ Jεometri, if yɷ fansi ɑ lįn az đε trak ov ɑ mɷviŋ pɵnt.

5. Ɑ plan iz ɑ flat surfas, lįk đε flɷr, er đε top ov đε tabl, er lįk yɷr slat. Ɨ ned not tel yɷ eni mɷr egzaktli hwot ɑ surfas iz, and hwot ɑ flat surfas menz ; bekez į am gɷiŋ tɯ be konfįnd tɯ wun plan fer ɑ loŋ hwįl. Ɨ mεn đat fer ɑ gɯd meni çapterz į

ʕ hwot pqrt ov đε strɷk ʃɯd đε lįn be. ʕHwot iz đε end ov ɑ lįn. ʕHɤ kan yɷ fansi đis sɷ az tɯ mak it lįk ɑ nedl-pɵnt. If ɑ pɵnt kɯd mɷv and lev ɑ trak behįnd it, ʕ hwot wɯd đat trak be. ʕHɤ ma el đat iz deskrįbd in đε fεrst ŧerti-fįv çapterz ov đε bɯk be drɑn.

ʃal tel yꝏ ꝏnli abȣt suç lįnz az kan be dren upon yꝏr slat er upon đe blakbꝏrd.

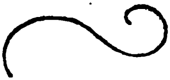
Nȣ yꝏ hav studid enuf fer wun leson. If yꝏ understand đis wel, yꝏ hav mad ɑ veri gɯd beginiŋ in Jeometri.

ÇAPTER III.

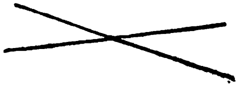
ABȢT STRƐT LĮNZ AND KURVZ.

1. Ɑ lįn đat iz not bent in eni pɋrt ov it iz keld ɑ strat lįn. If we fansi ɑ pơnt mꝏviŋ in ɑ strat lįn, we ʃal se it mꝏviŋ elwaz in đe sam direkʃon. Ɑ strat lįn iz đe ʃertest paŧ đat kan be mad from wun pơnt tɯ anuđer. Sꝏ, hwen we wiʃ tɯ tel đe distans from wun plas tɯ anuđer, we meʒɋr hȣ loŋ đe strat lįn iz đat jơnz đe tꝏ plasez. If ɑ ŧred iz streçt tįt akros ɑ tabl, it mɋrks ɑ strat lįn akros đe tabl. Đis iz đe wa đat kɋrpenterz mɋrk ɑ strat lįn, bį rubiŋ đe ŧred fęrst wiđ çek; and gɋrdnerz la ȣt gɋrden-pađz and bedz bį streçiŋ ɑ lįn.

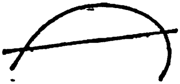
¶ Hwot is ɑ strat lįn. ¶ Hwot is đe direkʃon in hwiç ɑ pơnt mꝏvz, hwen mꝏviŋ in ɑ strat lįn. ¶ Hwot is đe ʃertest paŧ from wun plas tɯ anuđer. ¶ Hȣ duz ɑ kɋrpenter mɋrk ɑ strat lin. ¶ Hȣ duz ɑ gɋrdener mak hiz pađz strat. ¶ Hwot



2. Ɑ kurv lịn bendz in everi pɑrt, but haz nꝏ ʃɑrp kɵrnerz in it. And, if wɛ fansi ɑ pɵnt mꝏviŋ in ɑ kurv, wɛ ʃal sɛ it ɵl đɛ tịm çanjiŋ its direkʃon, but never takiŋ suden turnz. Perhaps it wil help yꝏ tu understand đɛ diferens betwɛn ɑ strat lịn and ɑ kurv, if ị drɵ tꝏ lịnz at đɛ sịdz ov đɛ paj. Ɨ ŧiŋk yꝏ wil understand, from hwot ị hav sed, hwiç iz đɛ strat lịn, and hwiç iz đɛ kurv.



3. Nȣ ị wont yꝏ tu sɛ đat tꝏ strat lịnz kan never kut akros ɛç uđer in mɷr đan wun plas. If yꝏ drɵ ɷnli tꝏ lịnz on yɷr slat, and ɛç lịn iz strat, đa kan not kros ɛç uđer in tꝏ plasez. But yꝏ kan not drɵ ɑ kurvd lịn đat yꝏ kan not kut, at lɛst, in tꝏ plasez. Trị tu drɵ, on yɷr slat, ɑ kurv đat kan not be kut in tꝏ plasez bị wun strat lịn.

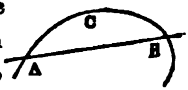


4. Đɑr iz wun ŧiŋ mɷr đat ị wont yꝏ tu lẹrn at đis

---

is ɑ kurv lịn. ⸮Hȣ dus ɑ pɵnt mꝏv in ɑ kurv. ⸮In hȣ meni plasez kan wun strat lịn kros anuđer. ⸮Kan ɑ strat lịn ɵlwas kut ɑ kurv in mɷr đan wun plas. ⸮In hȣ meni plasez kan ɑ strat lịn ɵlwas kut ɑ kurv. Let đɛ tẹçer drɵ ɑ kurv upon đɛ blakbɵrd, kros it bị ɑ strat lịn, and đen, mꝏviŋ đɛ çɵk aloŋ đɛ kurv, rɛkwịr đɛ skolɑrz tu sa "nȣ," hwenever đɛ çɵk iz mꝏviŋ in đɛ sam direkʃon as đɛ strat lịn.

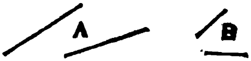
leson. Hwenever ɑ strat lịn jønz tꝏ pønts ov ɑ kurv, ɗạr iz əlwaz sum pønt on ɗɛ kurv betwen ɗɛ tꝏ pønts at hwiç ɗɛ kurv gꝏz in ɗɛ sam direkʃon az ɗɛ strat lịn. Sꝏ in mị figyụr yꝏ se ɗat ɗɛ kurv at c gꝏz in ɗɛ sam direkʃon az ɗɛ strat lịn A B. Dre on yꝏr slat figyụrz ov kurvz, and kut ɗem bị strat lịnz, and yꝏ wil fịnd it əlwaz sꝏ. Ɗis sɛmz lịk ɑ veri simpl ŧiŋ, and yet it iz ɑ veri yụsful trꝏŧ.



ȻAPTER IV.

ABƱT AŊGLZ.

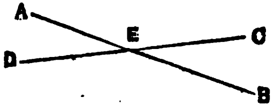
1. Hwen tꝏ strat lịnz gꝏ in diferent direkʃonz, ɗɛ diferens ov ɗạr direkʃonz iz kəld an aŋgl. Ɗɛ sịz ov ɗɛ aŋgl dɛpendz, ɗen, on ɗɛ diferens ov ɗɛ direkʃonz ov ɗɛ lịnz, and not on ɗạr leŋŧ. Sꝏ ɗat ɗɛ aŋgl hwiç Ị hav mạrkt B, iz lạrjer ɗan ɗat hwiç Ị hav mạrkt A, bekəz ɗạr iz mꝏr diferens in ɗɛ direkʃon ov ɗɛ tꝏ lịnz at B, ɗan ov ɗɛ tꝏ lịnz at A.



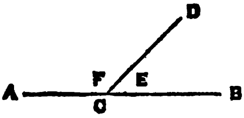
¿Hwot iz an aŋgl. ¿On hwot duz ɗɛ sịz ov an aŋgl depend. Let ɗɛ teçer dre aŋglz on ɗɛ blakbꝏrd, and ɑsk hwiç iz ɗɛ lạrjer, hwiç ɗɛ sməler, ets., beiŋ kạrful tꝏ mak sum ov ɗɛ

2. Đɛ pønt hwạr tɷ strat lịnz mɛt, ɵr hwạr đa wɯd mɛt if wɛ fansid đem drɵn loŋ ɛnuf, iz kɵld đɛ vẹrteks ov đɛ aŋgl. Đɛ vẹrteks ov đɛ aŋglz A and B iz not mạrkt dʊn; but yɷ ma drɵ tɷ strat lịnz mɛtiŋ, and đɛ pønt hwạr đa mɛt wil bɛ đɛ vẹrteks ov đɛ aŋgl betwɛn đem.

3. Hwen tɷ strat lịnz kros ɛç uđer, đa mak fωr aŋglz. Sω, in đɛ figyụr in đɛ mạrjin, wɛ hav tɷ strat lịnz makiŋ đɛ fωr aŋglz A E C, A E D, B E C, B E D. But hwen wɛ sa đa mak đɛ aŋgl A E C, wɛ hav tɯ fansi đɛ strat lịn D C gωiŋ in egzaktli đɛ opωzit direkʃon tɯ đat in hwiç it gωz tɯ mak đɛ aŋgl A E D.



4. If wɛ fansi đat đɛ lịn A B, in đɛ nekst figyụr, pønts from A tɯ B, hwịl đɛ lịn C D pønts from C tɯ D, đɛ tɷ lịnz wil mak đɛ aŋgl E. But if đɛ lịn A B pønts from B tɯ A, đa wil mak đɛ aŋgl F. Hwen đɛz tɷ aŋglz ạr ɛkwal, ɛç iz kɵld ɑ rịt aŋgl.



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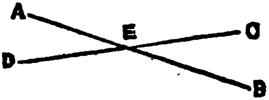
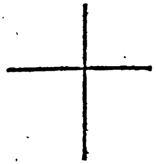
aŋglz widʊt vẹrtisez. ⸮ Hwot iz đɛ vẹrteks ov an aŋgl. Let đɛ teçer kɵl đɛ skolɑr tɯ đɛ bωrd tɯ pønt ʊt đɛ vẹrtisez ov đɛ aŋglz hɛ haz drɵn. Hwen tɷ strat lịnz kros ɛç uđer, ⸮hwot iz ɵlwaz trɷ abʊt đɛ aŋglz đa mak. ⸮ Hwot iz ɑ rịt aŋgl. ⸮ Hwot egzɑmplz kan yɷ giv ov ɑ rịt aŋgl. ⸮ Hwot iz đɛ

5. Hwen tω strat lįnz kros eç uđer, đɛ opωzit aŋglz ɋr ev đɛ sam sįz.



Ɨ mɛn đat in đis figyɥr, đɛ aŋgl A E C iz just az lɋrj az đɛ aŋgl D E B, and đɛ aŋgl A E D just az lɋrj az đɛ aŋgl C E B.

6. Hwen tω strat lįnz, krosiŋ eç uđer, mak fωr ɛkwal aŋglz, eç aŋgl iz keld ɑ rįt aŋgl. Drɵ tω lįnz on yωr slat at rįt aŋglz tɯ eç uđer.



Đɛ sįd ov yωr slat iz at rįt aŋglz tɯ đɛ botom. Đɛ top ov ɑ ʃɛt ov leter paper maks ɑ rįt aŋgl wiđ đɛ sįd. Đɛ sįdz ov everi skwąr kerner ɋr at rįt aŋglz tɯ eç uđer. Đɛ vẹrteks ov ɑ rįt aŋgl iz keld ɑ skwąr kerner. Hwen tω skwąr kernerz ɋr pωt tɯgeđer, đɛ ȣtsįd ejez wil ferm ɑ strat lįn.

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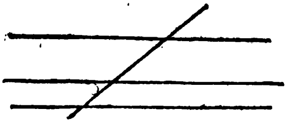
komon nam fer đɛ vẹrteks ov ɑ rįt aŋgl. Đɛ teçer must bɛ veri kąrfɯl not tɯ let đɛ çįld konfȣnd đɛ meʒɥr ov an aŋgl wiđ ɛđer đɛ leŋθ ov đɛ sįdz, er arɛɑ ov đɛ ωpniŋ betwɛn đem; but ilustrat and eksplan it ωnli bį diferensez ov direkʃonz; suç az pents ov kumpas, arωz, turniŋ yωr fas abȣt, ets. Hwen tω skwąr kerners ɋr pωt tɯgeđer, ¿ Hȣ dω đɛ ȣtsįdz run.

ꞒAPTER V.

PARALEL LĮNZ.

1. Hwen tɷ strat lịnz mak nɷ aŋgl wiđ ɛç uđer, ɵr hwen đa mak an aŋgl ɛkwal tɯ tɷ rịt aŋglz wiđ ɛç uđer, đa ɋr kɵld paralel. Đat iz tɯ sa, paralel lịnz ɋr strat lịnz đat pɷnt in đɛ sam direkʃon, ɵr in egzaktli opɷzit direkʃonz.



Trị hweđer yɷ kan drɵ suç upon yɷr slat.

2. Hwen tɷ strat lịnz ɋr paralel, đa ɋr just az fɋr apɋrt in wun plas az in anuđer. Đa kɯd not kum nɛrer and đen gɷ furđer apɋrt wiđɤt bendiŋ; but a strat lịn duz not bend in eni pɋrt. And, if đa kept kumiŋ nɛrer until đa met, đa wɯd mak an aŋgl wiđ ɛç uđer, and đɛ pɷnt hwɋr đa met wɯd bɛ đɛ vẹrteks ov đɛ aŋgl. But paralel lịnz mak nɷ aŋgl wiđ ɛç uđer.

3. If tɷ strat lịnz ɋr just az fɋr apɋrt in wun plas az in anuđer, đa ɋr paralel; đa run in đɛ sam

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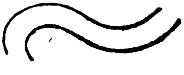
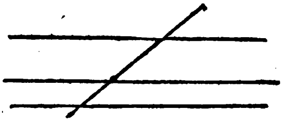
Lɯk abɤt đɛ rɷm, ɵr ɤt ov đɛ windɷ, and tel mɛ hwot strat lịnz yɷ kan sɛ. ʕDɷ yɷ sɛ eni kurv lịnz. ʕEni lịnz đat mak aŋglz wiđ ɛç uđer. ʕKan yɷ ʃɷ mɛ eni paralel lịnz. ʕEni konsentrik kurvz. ʕHwot ɋr paralel lịnz. ʕHwot kan direkʃon. Trį hweđer đɛ sįdz ov yꝏr slat ąr paralel, bį meʒųriŋ hweđer đa ąr just az fąr apąrt at đɛ top ov đɛ slat az at đɛ botom.

4. Hwen tꝏ kurvz ąr everihwąr at đɛ sam distans apąrt, đa ąr kɵld konsentrik kurvz. Sumtįmz đa ąr kɵld paralel kurvz; but đis iz not sꝏ gʊd ɑ nam fɵr đem az konsentrik kurvz.



5. Hwen ɑ strat lįn krosez tꝏ paralel lįnz, it maks đɛ sám aŋglz wiđ đɛ wun az wiđ đɛ uđer. Đɛ direkʃon ov paralel lįnz iz alįk, and sꝏ đɛ diferens ov đąr direkʃonz from đat ov đɛ strat lįn must be alįk.



6. If ɑ strat lįn iz paralel tɯ wun ov tꝏ paralel lįnz, it iz parelel tɯ đɛ uđer. Drɵ nȣ tꝏ paralel lįnz on yꝏr slat. Drɵ ɑ ŧẹrd lįn paralel tɯ wun ov yꝏr fẹrst, and it wil bɛ paralel tɯ đɛ uđer. Θl ŧrɛ ov đɛ lįnz wil pɷnt in đɛ sam direkʃon.

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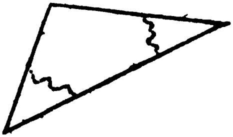
yꝏ sa abȣt đɛ distans apąrt ov paralel lįnz. ˁHwen ɑ strat lįn krosez tꝏ paralel lįnz, hwot kan yꝏ sa abȣt đɛ aŋglz. (Let đɛ teçer bɛwąr ov fɷrsiŋ ɑ çįld tɯ rɛpɛt đɛ rɛzniŋ ov sekʃonz tꝏ and fįv. Tɯ đɛ teçer đɛ rɛzniŋ iz ɛzier đan đɛ konsepʃonz; tɯ đɛ çįld it iz just đɛ revẹrs.) ˁHȣ kan yꝏ tel hweđer tꝏ lįnz ąr paralel.

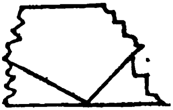
ȻAPTER VI.

Ɑ LITL ABƱT TRĮAŊGLZ.

1. Ɑ trįaŋgl iz ɑ figyųr bʊnded bį ŧrɛ strat lįnz. Đa ąr veri simpl-lɯkiŋ ŧiŋz; and yet đąr ąr meni kųrius ŧiŋz nωn abʊt đem əlredi. And đωz đat nω đɛ mωst abʊt Jɛometri tel us đat nω wun haz yet fʊnd ʊt əl đat kan bɛ nωn abʊt đem.



2. Đɛ ŧrɛ aŋglz ov ɑ trįaŋgl takn tɯgeđer wil mak tω rįt aŋglz. Yω kan trį it, if yω lįk, bį kutiŋ ɑ trįaŋgl ʊt ov paper wiđ ɑ pąr ov sizorz. Bɛ veri kąrful tɯ mak đɛ ejez strat. Nʊ kut of tω ov đɛ kərnerz bį ɑ waviŋ lįn, and la đɛ ŧrɛ kərnerz ov đɛ trįaŋgl kąrfuli tɯgeđer. Đɛ ʊter ejez wil mak wun strat lįn, just az if yω had put tω skwąr kərnerz tɯgeđer. Yω ma mak đɛ trįaŋgl ov eni ʃap ər sįz, and, if đɛ ejez ąr strat, yω wil əlwaz fįnd đat đɛ ŧrɛ kərnerz

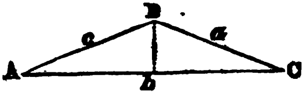


Drə trįaŋglz, and ɑsk "ʕHwot iz đɛ nam ov đez figyųrz." ʕHwot iz ɑ trįaŋgl. Drə ɑ rįt trįaŋgl, and, pʊntiŋ tɯ đɛ skwąr kərner, ɑsk, "ʕHwot aŋgl iz điz." ʕHʊ meni skwąr pɯt tɯgeđer mak a strat lɉn wiđ đąr ɤter ejez, just az tω skwąr kernerz wɯd dω. And đis iz hwot wɛ mɛn bɉ saiŋ đat đɛ ŧrɛ aŋglz ov ɑ trɉaŋgl takn tɯgeđer, wil mak tω rɉt aŋglz.

3. Ɑ trɉaŋgl kan not hav mωr đan wun aŋgl az ląrj az ɑ rɉt aŋgl.

4. If wun aŋgl in ɑ trɉaŋgl iz ɑ rɉt aŋgl, đɛ uđer tω, put tɯgeđer, wil, ov kωrs, just bɛ ɛkwal tɯ ɑ rɉt aŋgl. Yω kan trɉ đis bɉ kutiŋ paper trɉaŋglz wiđ wun skwąr kerner, and đen kutiŋ of đɛ uđer kernerz bɉ ɑ waviŋ lɉn, and pɯtiŋ đem tɯgeđer.

5. If wun sɉd ov ɑ trɉaŋgl iz loŋger đan anuđer sɉd, đɛ aŋgl opωzit đɛ loŋger sɉd iz ląrjer đan đat opωzit đɛ ʃerter sɉd. Nɤ lɯk at đɛ figyųr. Đɛ sɉd *a* iz opωzit đɛ aŋgl A, and đɛ sɉd *b* opωzit đɛ aŋgl B. Đɛ sɉd *b* iz loŋger đan đɛ sɉd *a*; and from đis wɛ ma nω đat đɛ aŋgl B iz ląrjer đan đɛ aŋgl A.



6. Nɤ, on đɛ uđer hand, hwen wun aŋgl in ɑ trɉaŋgl iz ląrjer đan anuđer, đɛ sɉd opωzit đɛ ląrjer aŋgl

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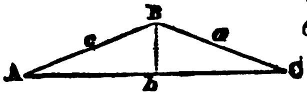
kernerz kan ɑ trɉaŋgl ever hav. ʕ Hɤ muq dω đɛ ŧrɛ aŋglz ov ɑ trɉaŋgl pɯt tɯgeđer mak. ʕ If wun aŋgl iz ɑ rɉt aŋgl, hɤ muq dω đɛ uđer tω put tɯgeđer mak. ʕ If wun sɉd ov ɑ trɉaŋgl iz loŋger đan anuđer, hwot dω yω nω abɤt đɛ aŋglz.

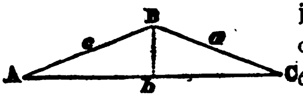
is loŋger đan đɛ sjd opωzit đɛ smeler aŋgl. Sω, if we nω đat B iz lɋrjer đan C, wɛ ma nω đat *b* iz loŋger đan *c*.



ƟAPTER VII.

MωR ABȢT TRƗAŊGLZ.

1. Supωz đat wɛ fȣnd tω sjdz ov ɑ trjaŋgl tɯ bɛ just ɛkwal tɯ ɛç uđer, ʕhwot ʃɯd wɛ nω abȣt đɛ aŋglz. Wɛ ʃɯd nω đat đɛ aŋgl opωzit wun sjd woz just az lɋrj az đɛ aŋgl opωzit đɛ uđer sjd. If đɛ sjd *a* iz just az loŋ az đɛ sjd *c*, đɛ aŋgl A iz just az lɋrj az đɛ aŋgl C.

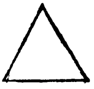


2. But if, on đɛ uđer hand, wɛ nω đat đɛ tω aŋglz ɋr ɛkwal, wɛ ʃal nω from đát đat đɛ tω sjdz ɋr ɛkwal. If đɛ aŋgl A iz just az lɋrj az đɛ aŋgl C, đen đɛ sjd *a* must bɛ just az loŋ az đɛ sjd *c*. Suç ɑ trjaŋgl iz keld jsóselez, hwiç menz ɛkwal-leged.

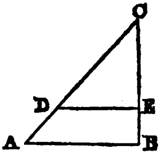
ʕ If wun aŋgl is lɋrjer đan anuđer, hwot dω yω nω abȣt đɛ sjdz.

Hwen wɛ nω đat tω sjdz ov ɑ trjaŋgl ɋr ɛkwal, ʕ hwot dω wɛ nω ov đɛ aŋglz. Hwen wɛ nω đat el đɛ sjdz ov ɑ trjaŋgl ɋr ov đɛ sam sjz, ʕ hwot dω wɛ nω abȣt đɛ aŋglz. Hwen wɛ

3. Nȣ if đɛ ŧrɛ sịdz ov ɑ triaŋgl ɋr ɛç ɛkwal tɯ ɛç uđɵr, đɛn đɛ aŋglz ɋr ɛkwal tɯ ɛç uđɵr; and if, òn đɛ uđɵr hand, đɛ ŧrɛ aŋglz ɋr ɛkwal tɯ ɛç uđɵr, đɛn đɛ sịdz ɋr ɛkwal tɯ ɛç uđɵr. Suç ɑ triaŋgl iz kɵld ɛkwiaŋgụlɑr, ɵr ɛkwilateral. Perhaps đis drɵiŋ wil help yɷ tɯ imajin an ɛkwilateral triaŋgl.



4. If ɑ lịn bɛ drɵn ŧrɷ ɑ triaŋgl paralel tɯ wun sịd ov đɛ triaŋgl, it divịdz đɛ uđɵr tɷ sịdz in đɛ sam prɷpɷrʃon. Ị mɛn đat, if in suç ɑ triaŋgl az A B C wɛ drɵ D E paralel tɯ A B, đɛn C D wil bɛ đɛ sam pɋrt ov C A đat C E iz ov C B. If C D iz tɷ-ŧẹrdz ov C A, đɛn C E wil bɛ tɷ-ŧẹrdz ov C B.



5. Bɛsịdz hwot ị hav ɵlrɛdi tɷld yɷ abȣt đɛ last figyụr,—đat iz, abȣt eni triaŋgl đat iz kut in tɷ bị ɑ lịn paralel tɯ wun sịd,—đɋr ɋr tɷ uđɵr kụrius ŧiŋz fɵr yɷ tɯ lẹrn. In đɛ fẹrst plas, A D wil bɛ in đɛ sam prɷpɷrʃon tɯ D C đat B E iz tɯ E C; and in đɛ sekond plas, C D wil bɛ in đɛ sam prɷpɷrʃon tɯ C E az A C iz tɯ B C, and A D wil ɵlsɷ bɛ in đɛ sam prɷpɷrʃon tɯ E B.

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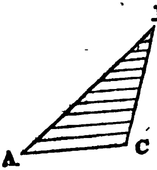
nɷ đat tɷ aŋglz ɋr ɛkwal, ʕ hwot dɷ wɛ nɷ abȣt đɛ sịds. ʕ Hwen wɛ nɷ đat đɛ ŧrɛ aŋglz ɋr ɛkwal. ʕ Hwot iz an ɛkwiaŋgụlɑr triaŋgl. ʕ Hwot iz an ɛkwilateral triaŋgl. Everi ɛkwiaŋgụlɑr triaŋgl iz ɵlsɷ ʕ——. And everi ɛkwilateral triaŋgl iz ɵlsɷ ʕ——. Let đɛ tɛçer nȣ kopi đɛ figyụr ov

If B C iz ŧre-kwerterz ov A C, đen C E wil be ŧre-kwerterz ov C D, and E B wil be ŧre-kwerterz ov A D.

6. If we divįd wun sįd ov ɑ trįaŋgl intɯ ɛkwal pąrts, and đen dre lįnz ŧrω đe pɵnts hwąr we hav divįded đe sįd, makiŋ đez lįnz paralel tɯ anuđer sįd ov đe trįaŋgl, đe ŧȩrd sįd wil be divįded intɯ ɛkwal pąrts. Đus, if đe sįd A B iz divįded intɯ ɛkwal pąrts, đe sįd B C iz ɵlsω đus divįded.



7. Ŧ am afrad đis leson wil be difikult tɯ understand; but in đe nekst į wil trį tɯ tel yω sumŧiŋ đat wil be ɛzier.

ÇAPTER VIII.

RĮT TRĮAŊGLZ.

1. If yω dω not remember hwot ɑ rįt aŋgl iz, yω must turn bak and rɛd çapter IV., sekʃon 6, and đen yω wil be redi tɯ gω on wiđ đis çapter.

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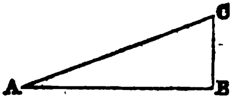
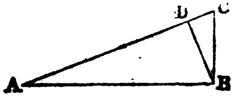
sekʃon fωr upon đɛ blakbωrd, and ɑsk ¿hwot lįnz ąr in đɛ sam prωpωrʃon az C D and C A. ¿Hwot in đɛ sam az C D tɯ C E. ¿Hwot in đɛ sam az A D tɯ D C. Let đɛ teçer, ɵlsω, dre paralel lįnz at ɛkwal distansez apąrt, lįk ɑ staf ov mųsik, and đen, streçiŋ ɑ striŋ akros đem, ʃω đɛ skolarz đat đɛ lįnz divįd đɛ striŋ ɛkwali in hwotever direkʃon it iz held.

Hwen ɑ trįaŋgl haz ɑ rįt aŋgl fɵr wun ov its aŋglz, it iz kɵld ɑ rįt trįaŋgl. Ɗɛ aŋgl A B C iz ɑ rįt aŋgl, and ɗɛ trįaŋgl A B C ɑ rįt trįaŋgl.

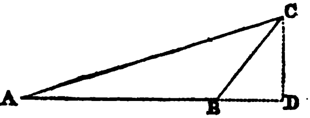


2. Eni trįaŋgl ma bɛ divįded intɯ tɷ rįt trįaŋglz, bį drɵiŋ ɑ lįn ŧrɷ ɗɛ vẹrteks ov ɗɛ lɑrjest aŋgl in suç ɑ wa az tɯ mak rįt aŋglz wiɗ ɗɛ loŋgest sįd. Ɗus, if B iz ɗɛ lɑrjest aŋgl in ɗɛ trįaŋgl A B C, wɛ kan drɵ B D in suç ɑ wa az tɯ mak ɗɛ aŋglz at D rįt aŋglz; and ɗis wil divįd ɗɛ trįaŋgl intɯ tɷ rįt trįaŋglz, A D B and C D B.



Nȣ drɵ eni trįaŋglz yɷ plɛz, upon yɷr slat, and trį hweɗer yɷ kan not ɵlwaz divįd ɗem, in ɗis wa, intɯ tɷ rįt trįaŋglz.

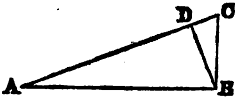
3. If ɗɛ lɑrjest aŋgl in ɑ trįaŋgl iz lɑrjer ɗan ɑ rįt aŋgl, wɛ kan ɵlwaz fansi ɑ rįt trįaŋgl aded tɯ it in suç ɑ wa az tɯ mak ɗɛ hωl figyụr ɑ rįt trįaŋgl. If B bɛ lɑrjer ɗan ɑ rįt aŋgl, wɛ kan mak A B loŋger, and drɵ C D dȣn in suç ɑ wa az tɯ



ʕ Hwot iz ɑ rįt aŋgl. ʕ Hwot iz ɗɛ komon nam fɵr ɗɛ vẹrteks ov ɑ rįt aŋgl. ʕ Hwot iz ɑ rįt trįaŋgl. ʕ Hȣ kan yɷ divįd eni trįaŋgl intɯ tɷ rįt trįaŋglz. Ɗɛ teçer ma kɵl ɗɛ klas tɯ mak D ɑ rịt aŋgl. Đen đɛ aded trịaŋgl B C D iz ɑ rịt trịaŋgl, and đɛ hɷl figyųr A D C iz ɛlsɷ ɑ rịt trịaŋgl. And, bị takiŋ awa B D C from A D C, wɛ ʃal hav A B C left. Sɷ đat eni trịaŋgl wiđ wun aŋgl lqrjer đan ɑ rịt aŋgl, lịk A B C, ma bɛ fansid az đɛ diferens betwɛn tɷ rịt trịaŋglz, lịk A D C and B D C.

4. If wɛ divịd ɑ rịt trịaŋgl intɯ tɷ rịt trịaŋglz, az ị hav tɷld yɷ hȣ tɯ dɷ in đɛ sekond sekʃon ov đis çapter, đɛ tɷ litl trịaŋglz wil bɛ ov egzakŧli đɛ sam ʃap az đɛ hɷl lqrj trịaŋgl.

Đus, if B iz ɑ rịt aŋgl, and đɛ aŋglz at D qr bɷŧ rịt aŋglz, đen đɛ ŧrɛ trịaŋglz A B C, A B D and B D C, qr ɛl ov egzakŧli đɛ sam ʃap.



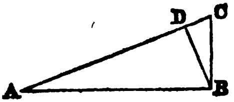
đɛ blakbɷrd, and alȣ đem tɯ drɛ trịaŋglz and divịd đem in đis wa. ʕTɯ hwot kịnd ov trịaŋgl kan yɷ ad ɑ rịt trịaŋgl sɷ az tɯ mak đɛ hɷl figyųr ɑ rịt trịaŋgl. Let đɛ pųpilz ʃɷ đis bị drɛiŋ đem on đɛ blakbɷrd. ʕHwot kịnd ov trịaŋgl kan bɛ fansid az đɛ diferens betwɛn tɷ rịt trịaŋglz. If wɛ divịd ɑ rịt trịaŋgl intɯ tɷ rịt trịaŋglz, ʕhwot dɷ yɷ nɷ abȣt đem.

ÇAPTER IX.

SIMILARITI AND ƗSƟPƐRIMƐTRI.

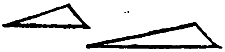
1. Đąr qr tɷ veri hqrd-lɯkiŋ wurdz at đɛ hed ov đis çapter; but đɑ qr not hqrd tɯ understand. Similariti mɛnz lįknes; and in Jɛometri it mɛnz đɛ haviŋ đɛ sam ʃap. Ɨsɷperímɛtri mɛnz đɛ beiŋ ov đɛ sam sįz rʊnd abʊt.

2. Hwen tɷ bodiz, ɵr tɷ jɛɷmetrikal figyụrz, qr ov egzaktli đɛ sam ʃap, wɛ kɵl đem similɑr. Hwen tɷ figyụrz qr similɑr, đat iz, ov egzaktli đɛ sam ʃap, đɛ aŋglz ov wun figyụr qr egzaktli ɛkwal tɯ đɛ aŋglz ov đɛ uđer figyụr, and ɵl đɛ sįdz ov đɛ fẹrst figyụr qr in đɛ sam prɷpɷrʃon tɯ đɛ korespondiŋ sįdz ov đɛ uđer figyụr. Ɨ tɷld yɷ đat ADB and CDB qr ov đɛ sam ʃap. Sɷ đat đɛ aŋgl at A iz just az lqrj az đɛ aŋgl DBC, đɛ aŋgl at C just az lqrj az DBA, and đɛ tɷ aŋglz at D qr ɛkwal. And CB iz đɛ sam pqrt ov AB đat CD iz ov DB, ɵr đat DB iz ov AD. Ɵl đis iz

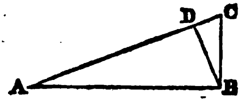


ꝰ Hwot iz đɛ mɛniŋ ov "similɑr" figyụrz. ꝰ Hwot iz trɷ ov đɛ aŋglz ov similɑr figyụrz. Let đɛ teçer drɵ tɷ similɑr kwodrilateralz on đɛ bɷrd, mak ɑ litl sẹrkl, stqr, kros and aksent, in đɛ fɷr aŋglz ov wun, and bid ɑ member ov đɛ klas ment bị saiŋ đat đɛ trịaŋgl C D B iz ov đɛ sam ʃap az A D B.

3. If we fịnd đat đɛ ŧre aŋglz ov wun trịaŋgl qr just ɛkwal tɯ đɛ ŧre aŋglz ov anuđer trịaŋgl, we ma nɷ đat đɛ ŧre sịdz ov đɛ fẹrst trịaŋgl qr in đɛ sam prɷpɷrʃon tɯ đɛ koresspondiŋ sịdz ov đɛ sekond trịaŋgl, and đat đɛ trịaŋglz qr similar. Đat iz tɯ sa, đat if đɛ ŧre aŋglz ov wun trịaŋgl qr ɛkwal tɯ đɛ ŧre aŋglz ov anuđer trịaŋgl, đɛ sịdz opɷzit tɯ đɛ ɛkwal aŋglz qr in đɛ sam prɷpɷrʃon tɯ ɛq uđer.



4. It iz elsɷ trɷ đat hwen tɷ figyqrz qr similar, eni tɷ sịdz ov đɛ wun qr in đɛ sam prɷpɷrʃon tɯ ɛq uđer đat đɛ korespondiŋ sịdz ov đɛ uđer qr in tɯ ɛq uđer. Đus, C D iz đɛ sam pqrt ov C B đat B D iz ov B A. And D B iz đɛ sam pqrt ov D A đat D C iz ov D B.



5. Θl đɛ nịs kalkqlaʃonz ov enjinerz, and maʃenists, and ʃip-bilderz, and navigatorz, and astronɷmerz, qr mad bị help ov similariti ov trịaŋglz.

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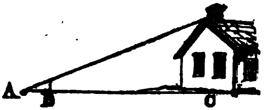
kum and put korespondiŋ mqrks in đɛ aŋglz ov đɛ uđer figyqr, đus: "ʕHɷ wil mqrk in đat figyqr đɛ aŋgl hwiq iz ɛkwal tɯ đis đat ị mqrkt wiđ a kros. ʕHɷ wil mqrk đɛ wun ɛkwal tɯ đis wun mqrkt wiđ a sẹrkl," ets., ets. Đen let đɛ teqer

Ɨ wil trị tɯ eksplan tɯ yꝏ wun siŋgl instans in hwiç yꝏ kan yɥz similariti ov trịaŋglz. Supɷz đat ɑ hʊs standz on level grʊnd, and yꝏ wiʃ tɯ fịnd ʊt hʊ hị it iz. Pɯt ɑ stak uprịt in đɛ grʊnd, enihwąr đat yꝏ ŧiŋk best, sa at B. Đen la yꝏr hed klꝏs tɯ đɛ grʊnd, and mꝏv it until yꝏ kan just sɛ đɛ top ov đɛ hʊs ɷver đɛ top ov đɛ stak. Đen meʒɥr hʊ fąr yꝏr ị iz from đɛ botom ov đɛ stak, and hʊ fąr from đɛ botom ov đɛ hʊs. Đen, az yꝏ wil sɛ bị đɛ figyɥr, yꝏ wil hav tꝏ trịanglz ov đɛ sam ʃap, and đɛ hịt ov đɛ hʊs wil be đɛ sam pąrt ov đɛ distans A C, đat đɛ hịt ov đɛ stak iz ov đɛ distans A B. If đɛ hịt ov đɛ stak iz ɛkwal tɯ đɛ distans ov yꝏr ị from đɛ botom ov it, đen đɛ hịt ov đɛ hʊs iz just ɛkwal tɯ đɛ distans ov yꝏr ị from đɛ fʊndaʃon ov đɛ hʊs. Anuđer wa ov fịndiŋ similar trịaŋglz tɯ meʒɥr ɑ hʊs, ɛr trɛ, on level land, iz bị yɥziŋ ʃadɷz. Đɛ ʃadɷ ov đɛ stak B, hwen đɛ sun ʃɷn, wɯd be ɑ trịaŋgl. Đɛ stak wɯd be wun sịd, đɛ ʃadɷ on đɛ grʊnd anuđer, and đɛ ŧɛrd sịd wɯd be đɛ ej ov đɛ ʃadɷ in đɛ ąr, gɷiŋ from đɛ top ov đɛ stak tɯ đɛ end ov đɛ ʃadɷ on đɛ grʊnd. Ɑ similar trịaŋgl wɯd, at đɛ sam



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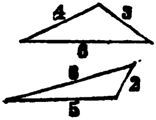
mak ɑ kros ŧrꝏ tꝏ sịdz ov wun kwodrilateral, and sa, "ʕ Hꝏ wil mąrk đɛ tꝏ sịdz in đɛ uđer figyɥr đat ąr in đɛ sam prɷpɷrʃon az đɛz." Nekst let him dɪɵ ɑ figyɥr ov ɑ liberti-pɷl, and tįm, bɛ mad bį đɛ ʃadω ov đɛ trɛ er hȣs. Sω đat, if yω ʃɯd meʒɥr at eni tįm đɛ leŋϑ ov đɛ ʃadω ov đɛ stak, and đɛ leŋϑ ov đɛ ʃadω ov đɛ hȣs, yω kɯd tel đɛ hįt ov đɛ hȣs; bekez đɛ hįt ov đɛ hȣs wɯd bɛ đɛ sam pqrt ov đɛ leŋϑ ov its ʃadω đat đɛ stak woz ov its ʃadω.

6. Hwen it iz just az fqr rȣnd wun figyɥr az it iz rȣnd anuđer, đɛ tω figyɥrz qr keld įsωperimétrikal, hwiç mɛnz, ɛkwal rȣnd abȣt.



If wun trįaŋgl meʒɥrz, on its sįdz, tω fɛt, and fįv fɛt, and siks fɛt, its perímeter, đat iz, đɛ distans rȣnd it, wil bɛ ϑɛrten fɛt; bekez tω and fįv and siks mak ϑɛrten. And if anuđer trįaŋgl meʒɥrz on its sįdz, ϑrɛ fɛt, and siks fɛt, and fωr fɛt, its perimeter wil elsω bɛ ϑɛrten fɛt; bekez ϑrɛ and siks and fωr mak ϑɛrten. Sω, đɛz tω trįaŋglz wil bɛ įsωperimetrikal; but đa wil not bɛ similar, đat iz, đa wil bɛ ov diferent ʃaps.

7. Similar figyɥrz qr đωz ov đɛ sam ʃap; įsωperimetrikal figyɥrz qr đωz hwiç meʒɥr ɛkwali rȣnd abȣt.

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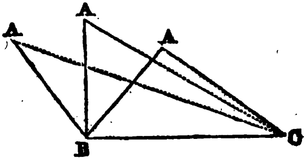
a stak, and ask đɛ çįld tω eksplan in hwot wa hɛ kan meʒɥr đɛ hįt ov đɛ liberti-pωl bį mɛnz ov đɛ stak. ¶ Hwen qr tω figyɥrz keld įsωperimetrikal.

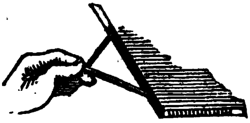
ꞒAPTER X.

ĐƐ SĪZ OV TRĪAŊGLZ.

1. If wun sįd ov ɑ trįaŋgl kan grꝏ loŋger ɵr ʃɵrter, hwįl đɛ opꝏzit aŋgl ꝏpnz and ʃuts, az dꝏ its vẹrteks wẹr ɑ hinj, đɛ trįaŋgl wil bɛ lɋrjest hwen đat aŋgl iz ɑ rįt aŋgl. Đus, if A B and B C kan not bɛ çanjd in leŋŧ, but if A C iz lįk an Indiɑruber kɵrd, and kan bɛ mad loŋger ɵr ʃɵrter bį ɵlteriŋ đɛ aŋgl at B, đen đɛ trįaŋgl A B C wil bɛ lɋrjest hwen đɛ aŋgl at B iz ɑ rįt aŋgl.

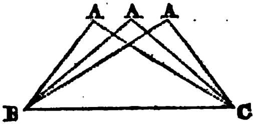


2. Yꝏ kan ʃꝏ đis veri pritili in đis wa. Tak tꝏ litl strat stiks and hꝏld đem tɯgeđer at wun end wiđ yꝏr ŧum and fiŋger, hwįl yꝏ spred đɛ uđer tꝏ endz agenst đɛ ej ov đɛ tabl. Đɛ trįaŋgl mad bį đɛ tꝏ stiks and đɛ tabl-ej wil bɛ lɋrjest hwen đɛ stiks mak ɑ rįt aŋgl wiđ ɛç uđer.

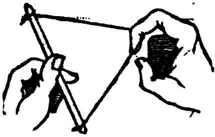


If tꝏ stiks lɛn đɋr tops tɯgeđer, ʕhɤ must đa bɛ plast tɯ mak đɛ spas bɛtwɛn đem and đɛ grɤnd lɋrjest. If tꝏ bꝏrds

3. If wun sįd ov ɑ trįaŋgl kan not çanj in leŋŧ, and if đɛ uđer tω kan ωnli çanj in suç ɑ wá az tɯ kɛp đɛ trįaŋgl įsωperimetrikal, đɛ trįaŋgl wil bɛ ląrjest hwen đɛz tω sįdz ąr ov ɛkwal leŋŧ. Đus, if B C kan not grω ɛđer loŋger ər ʃərter, and if A B grωz ʃərter egzaktli az fąst az A C grωz loŋger, ər grωz loŋger egzaktli az fąst az A C grωz ʃərter, đɛ trįaŋgl wil bɛ ląrjest hwen A B iz ɛkwal tɯ A C.



4. Yω ma ʃω đis in ɑ veri priti wa bį tįiŋ đɛ endz ov ɑ striŋ tɯ đɛ endz ov ɑ strat stik ɑ gɯd dɛl ʃərter đan đɛ striŋ. Đen tak đɛ stik in wun hand, and, pɯtiŋ wun fiŋger ov đɛ uđer insįd đɛ striŋ, pɯl it tįt. Đɛ trįaŋgl fərmd betwɛn đɛ striŋ and đɛ stik wil bɛ ląrjest hwen yωr fiŋger iz in đɛ midl ov đɛ striŋ. But hwen yω mωv yωr fiŋger đɛ trįaŋgl rɛmanz įsωperimetrikal, if đɛ striŋ duz not streç.



5. If wɛ supωz đat əl ŧrɛ sįdz ov ɑ trįaŋgl kan çanj đąr leŋŧ, but ωnli in suç ɑ wa az tɯ kɛp đɛ trįaŋgl įsωperimetrikal, đen đɛ trįaŋgl wil bɛ ląrjest

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bɛ nald tɯgeđer tɯ mak ɑ pig-trəf, ʕhwot aŋgl must đa mak sω as tɯ hav đɛ trəf hωld mωst. If ɑ tent is mad simpli ov tω slant sįdz lįk đɛ rωf ov ɑ hɤs, ʕhɤ must wɛ piç it sω as tɯ hav mωst rωm in it.

hwen đɛ ŧrɛ sȷ̣dz ɋr ɛkwilateral. Đat iz tɯ sa, đat an ɛkwilateral trȷ̣aŋgl iz đɛ lɋrjest amuŋ ȷ̣sωperimetrikal trȷ̣aŋglz.

6. Yꝏ kan ʃω đis bȷ̣ takiŋ ɑ loŋ pɛs ov striŋ and tȷ̣iŋ đɛ endz tɯgeđer. Put intɯ đis lꝏp wun fiŋger ov yꝏr rȷ̣t hand, and get tꝏ plamats ɛç tɯ dꝏ đɛ sam. Stepiŋ bak, yꝏ kan pul đɛ striŋ intɯ ɑ lɋrj trȷ̣aŋgl, and yꝏ wil sɛ đat it iz lɋrjest hwen yꝏr fiŋgerz ɋr at ɛkwal distansez, and đɛ trȷ̣aŋgl ɛkwilateral. But ɵl đɛ trȷ̣aŋglz mad bȷ̣ mꝏviŋ đɛ fiŋgerz wil bɛ ȷ̣sωperimetrikal, bekɵz đɛ striŋ remanz ov wun leŋŧ. Hwen yꝏ grω ωlder yꝏ kan prꝏv đɛz ŧiŋz; but nɤ yꝏ kan best ʃω đem tɯ yꝏrself in suç waz az đis.

ꞒAPTER XI.

DIFERENT KƗNDZ OV TRƗAŊGLZ.

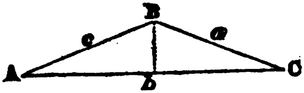
1. Hwen ɑ trȷ̣aŋgl haz its ŧrɛ sȷ̣dz ɛkwal, it iz kɵld an ɛkwilateral trȷ̣aŋgl. Ɛkwilateral mɛnz ɛkwal-sȷ̣ded. Everi ɛkwilateral trȷ̣aŋgl haz its aŋglz ɛkwal tɯ ɛç uđer; and iz, đɋrfωr, kɵld ɛkwiaŋgɥlɑr. But ɛkwilateral iz đɛ mωr komon nam.

2. Hwen ɑ trȷ̣aŋgl haz tꝏ ov its sȷ̣dz ov đɛ sam leŋŧ it iz kɵld an ȷ̣sósɛlez trȷ̣aŋgl. Ɨsosɛlez mɛnz

ɛkwal-legd. Ðe aŋglz opꝏzit ðe ɛkwal legz ɑr, az ị hꝏp yꝏ remember, ɛkwal tɯ ɛç uðer.

An ɛkwilateral trịaŋgl must be ịsoselez; but an ịsoselez trịaŋgl ma not be ɛkwilateral. Everi hers iz an animal; but not everi animal iz ɑ hers.

3. Supꝏz ðat A B C iz an ịsoselez trịaŋgl, and ðat *a* iz ɛkwal tɯ *c*. Nȣ, if we dre ɑ lịn B *b* from B tɯ ðe midl ov A C, it wil divịd ðe trịaŋgl intɯ tꝏ ɛkwal rịt trịaŋglz. Ðe aŋgl at B iz divịded egzaktli in ðe midl, ðe aŋglz at *b* ɑr rịt aŋglz, and we kud fꝏld ðe trịaŋgl ꝏver on ðe lịn B *b* az ɑ hinj, and ðe tꝏ pesez wud fit ɛç uðer egzaktli.



4. If ɑ trịaŋgl haz nꝏ tꝏ sịdz ɛkwal, it iz keld ɑ skalen trịaŋgl. Skalen menz lam, er limpiŋ.

5. Hwen ɑ trịaŋgl haz wun rịt aŋgl, it iz, az ị hꝏp yꝏ remember, keld ɑ rịt trịaŋgl. We kan hav ịsoselez rịt trịaŋglz, and skalen rịt trịaŋglz; but, ov kꝏrs, we kan not hav ɛkwilateral rịt trịaŋglz.

Let ðe teçer prꝏvịd stiks and strips and mak trịaŋglz, as direkted in sekʃonz tꝏ, fꝏr and siks, and ɑsk, "Hwen wil ðis trịaŋgl be lɑrjest."

Set ðe klas at ðe blakbꝏrd, and ɑsk ðem tɯ dre an ɛkwilateral trịaŋgl, and tel hwot ða nꝏ abȣt its aŋglz and sịdz. An ịsoselez trịaŋgl. Ɑ skalen trịaŋgl. An ịsoseles oblek trị-

6. Hwen neđer ov đɛ aŋglz ov ɑ trịaŋgl iz ɑ rịt aŋgl, đɛ trịaŋgl iz keld an oblɛ́k trịaŋgl. Oblɛk trịaŋglz ma bɛ ɛkwilateral, er ịsosɛlɛz, er skalɛn.

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aŋgl. Ịsosɛlɛz rịt trịaŋgl. Skalɛn rịt trịaŋgl. Skalɛn oblɛk trịaŋgl. Ɑsk đem, elsɷ, tɯ pɵnt ɤt el đɛ trịaŋglz in objekts wiđin sịt, and nam đem akerdiŋ tɯ đis çapter.

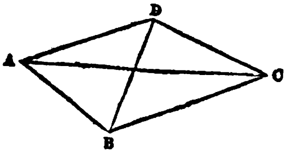
Revų ov Trịaŋgls.—Let đɛ skolɑrz nɤ turn bak tɯ çapter VI., and tak siks çapterz as ɑ rɛvų-leson, and ɑnser đɛ folɷiŋ kwestyonz, and uđerz sɛlekted from đɛ kwestyonz on ɛç çapter.

ʕHwot is ɑ trịaŋgl. If đɛ ŧrɛ kernerz ov ɑ paper trịaŋgl qr plast tɯgeđer, ʕhwot wil đqr sum bɛ. ʕHɤ wil đɛ ɤter ejez run. In ɑ rịt trịaŋgl ʕhwot wil đɛ sum ov đɛ tɷ smeler aŋglz bɛ. In an ịsosɛlɛz rịt trịaŋgl ʕhwot pqrt ov ɑ rịt aŋgl wil ɛç smeler aŋgl bɛ. ʕHɤ qr đɛ smelest aŋgl in ɑ trịaŋgl and đɛ ʃertest sịd in đɛ sam trịaŋgl plast. If wun kerner ov ɑ trịaŋgl bɛ kut of bị ɑ lịn paralel tɯ đɛ opɷzit sịd, ʕhwot iz đɛ ʃap ov đɛ litl trịaŋgl đat yɷ đus mak. If wɛ divịd ɑ rịt trịaŋgl intɯ tɷ rịt trịaŋglz bị ɑ lịn ŧrɷ đɛ vẹrteks ov đɛ rịt aŋgl, ʕhwot wil bɛ đɛ ʃap ov đɛz litl trịaŋglz. ʕHwen qr tɷ trịaŋglz similɑr. ʕHwen ịsɷperimetrikal. Hwen ɷnli wun sịd ov ɑ trịaŋgl kan elter in leŋŧ, ʕhɤ ʃal wɛ mak đɛ trịaŋgl lqrjest. Hwen tɷ sịdz ɷnli kan elter, but đɛ trịaŋgl iz kept ịsɷperimetrikal, ʕhɤ iz it mad lqrjest. ʕHwot iz đɛ lqrjest ov el ịsɷperimetrikal trịaŋglz.

ÇAPTER XII.

KWODRAŊGLZ.

1. Hwen ɑ figyųr iz bɜnded bị fɷr strat lịnz, it iz keld ɑ kwodraŋgl. Sumtịmz it iz keld ɑ kwodrilateral, but jenerali ɑ kwodraŋgl.



2. Ɑ lịn jơniŋ tꝏ opɷzit vęrtisez iz keld ɑ dịagɷnal. In ɗis figyųr, ɗɛ doted lịn A C iz ɑ dịagɷnal- and ɗɛ doted lịn B D iz anuɗer dịagɷnal.

3. Ɛç dịagɷnal divịdz ɗɛ kwodraŋgl intɯ tꝏ trịaŋglz. If boŧ dịagɷnalz ɑr drən, ɗɛ kwodraŋgl iz kut up intɯ fɷr litl trịaŋglz ; but ɗɷz ɑr not ɗɛ trịaŋglz ov hwiç ị am spekiŋ. Ɗɛ doted lịn A C, in ɗɛ figyųr, divịdz ɗɛ kwodraŋgl intɯ tꝏ trịaŋglz, A B C and C D A. And ị ŧiŋk yꝏ kan ɛzili se ɗat, if we pɯt el ɗɛ kernerz ov ɑ kwodraŋgl tɯgeɗer, ɗa wil mak

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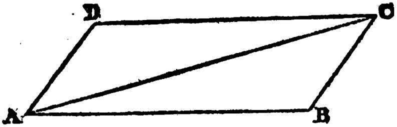
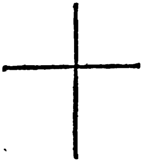
¿Hwot iz ɑ kwodraŋgl. ¿Hwot dꝏ yꝏ kol ɑ lịn ɗat jơnz tꝏ opɷzit vęrtisez. ¿Intɯ hwot tꝏ figyųrz iz ɑ kwodraŋgl divịded bị ɑ dịagɷnal. ¿Hwot iz ɗɛ sum ov ɗɛ aŋglz ov ɑ trịaŋgl. ¿Hwot iz ɗɛ sum ov ɗɛ aŋglz ov ɑ kwodraŋgl. ¿Hɜ kan yꝏ ʃɷ ɗis bị paper figyųrz. Let, nɜ,

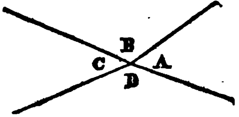


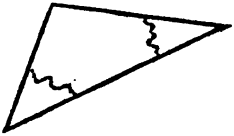
just az lạrj ɑ sum az if wɛ put el đɛ aŋglz ov tꝏ trịaŋglz tɯgeđer.



4. Đɛ sum ov đɛ fꝍr aŋglz ov eni kwodraŋgl iz ɛkwal tɯ fꝍr rịt aŋglz. But if wɛ put đɛ vẹrtisez ov fꝍr rịt aŋglz tɯgeđer, đɛ sịdz ov đɛ aŋglz wil mak tꝏ strat lịnz krosiŋ ɛç uđer.

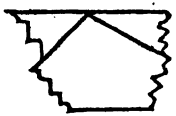
And if wɛ mak ɑ paper kwodraŋgl, kut of đɛ kerṇerz (bị ɑ waviŋ lịn, sꝍ az not tɯ bɛkum konfɥzd), and put đɛz fꝍr kernerz tɯgeđer, wɛ ʃal fịnd đat đɑ fil up el đɛ spas arȣnd đɛ kernerz. Kompąr nȣ đɛ leterz in đɛz fꝍr aŋglz wiđ đɛ leterz at đɛ kerner ov đɛ fẹrst kwodraŋgl đat ị drꝏ fer yꝏ, on paj 34.





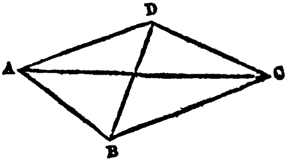
In kutiŋ paper trịaŋglz and kwodraŋglz, tɯ ʃꝍ đat đɛ kernerz put tɯgeđer mak tꝏ er fꝍr rịt aŋglz, yꝏ

đɛ tɛçer drɛ kwodraŋglz and dịagꝍnalz on đɛ bꝍrd, and ɑsk, "¿ Hwot iz đis figyɥr. ¿ Hwot iz đis lịn," ets. ¿ Hwot iz đɛ

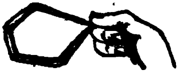


must be kąrful tu mak đɛ sįdz strat. Kut of đɛ kernerz bį ɑ wavin lįn, đat yɷ ma distingwiʃ đɛ ejez ov đɛ kuts from đɛ ejez ov đɛ trįanglz ɵr kwodranglz.

5. Đɛ ʃap ov ɑ trįangl kan not be ɵlterd, eksept bį ɵlterin đɛ lenŧ ov at lɛst wun sįd. But đɛ ʃap ov ɑ kwodrangl kan be ɵlterd wiđʊt ɵlterin đɛ lenŧ ov ɑ singl sįd, bį ɵlterin đɛ anglz. Đɛ dįagɷnalz ov ɑ kwodrangl kan be mad ʃɵrter ɵr longer wiđʊt ʃɵrtnin ɵr lenŧenin đɛ sįdz. If yɷ wil luk at đɛ figyųr yɷ wil se đat B D mįt be puld ʊt, and A C krʊded tugeđer, sɷ az tu ɵlter đɛ ʃap ov đɛ kwodrangl veri muç, wiđʊt ɵlterin đɛ lenŧ ov its sįdz at ɵl.

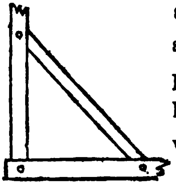


6. Tak tɷ wilɷ twigz, ɵr tɷ ŧin rɷlz ov paper, suç az ąr yųzd fɵr lamp-lįterz, and bend wun intu ɑ trįangl, and wun intu ɑ kwodrangl. Yɷ wil fįnd đat đɛ trįangl iz stif, and đat đɛ kwodrangl iz not.



ɵnli figyųr đat iz stron and stif. ʕ In hwot figyųrz kan đɛ anglz be ɵlterd wiđʊt ɵlterin đɛ sįdz. ʕ Hwot iz đɛ yųs ov ɑ bras. Let đɛ teçer prɵvįd twigz ɵr lamp-lįterz fɵr đɛ ilustraʃon ov sekʃon siks.

7. If yω hav ever nωtist đɛ fram ov ɑ hȣs, yω hav sen đat đɛ kɑrpenter pɯts brasez in đɛ kɵrnerz. Wiđȣt brasez đɛ timberz wɯd ωnli mak kwodraŋglz and sω wɯd hav nω stifnes eksept đɛ stifnes ov jɵnts. But, bị pɯtiŋ brasez hɛ maks trịaŋglz, hwiç kan not bɛ prest ȣt ov ʃap wiđȣt beiŋ brωkn.

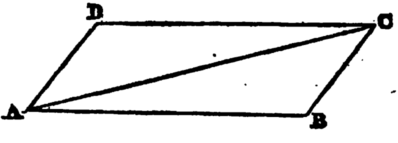


ȻAPTER XIII.

PARALELωGRAMZ.

1. Hwen ɑ kwodraŋgl haz its opωzit sịdz paralel it iz kɵld ɑ paralelωgram. Sω đat đɑr ɑr in everi paralelωgram tω sets ov paralel sịdz.

2. Đɛ opωzit aŋglz ov ɑ paralelωgram ɑr ɛkwal. If đis figyɥr iz ɑ paralelωgram, sω đat AD iz paralel

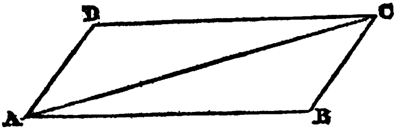


¿Hwot iz ɑ paralelωgram. ¿Hȣ meni sets ov paralel sịds in ɑ paralelωgram. ¿Hȣ meni sets ov ɛkwal aŋglz. ¿Hwiç tɯ B C, and D C paralel tɯ A B, đen đɛ aŋgl at D iz ɛkwal tɯ đɛ aŋgl at B, and đɛ aŋgl at A iz ɛkwal tɯ đat at C.

3. Ɑ dịagɷnal divịdz ɑ paralelɷgram intɯ tꝏ ɛkwal trịaŋglz. If yꝏ kɯd turn đɛ trịaŋgl A B C rʊnd, sɷ az tɯ pɯt đɛ pɵnt B egzaktli on đɛ pɵnt D, and đɛ lịn B C upon đɛ lịn D A, đen đɛ lịn B A wɯd lị upon đɛ lịn D C, and wun trịaŋgl wɯd egzaktli kuver đɛ uđer. Yꝏ kan trị đis bị kutiŋ ɑ paralelɷgram ov paper, and kutiŋ it in tꝏ, dịagɷnali. But yꝏ must be veri kạrfɯl tɯ hav đɛ opɷzit sịdz egzaktli paralel.

4. And đis ʃɷz yꝏ đat in everi paralelɷgram đɛ opɷzit sịdz ạr ɛkwal. In đɛ figyụr, A D iz ɛkwal tɯ B C, and A B tɯ D C. If đɛ opɷzit sịdz ov ɑ kwodraŋgl ạr paralel, đa ạr ɛkwal.

5. And if ɑ kwodraŋgl haz its opɷzit sịdz ɛkwal, đa ạr elsɷ paralel. If ị ʃɯd fịnd ɑ kwodraŋgl suq az đis figyụr, and, bị meʒụriŋ, fịnd đat A D and B C



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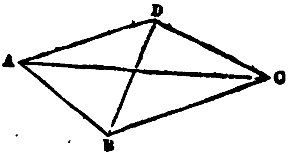
aŋglz ov ɑ paralelɷgram ạr ɛkwal. Let đɛ klas dre paralelɷgramz on đɛ blakbɷrd, and ʃɷ hwiq aŋglz ạr ɛkwal. Dre ɑ dịagɷnal in yꝏr paralelɷgram ⸮Hʊ duz it divịd đɛ figyụr. ⸮Hʊ kan yꝏ tel hweđer ɑ figyụr iz ɑ paralolɷgram. ⸮Hʊ qr just ɛkwal, and ɵlsꝏ ðat A B and D O qr ɛkwal tɯ eç uðer, ị ʃud nꝏ ðat ðez ɛkwal lịnz qr paralel.

6. Hwen ɑ kwodraŋgl iz skwezd flater wiðɜt ɵltering ðɛ leŋθ ov its sịdz, it ɵlterz ðɛ sịz, az wel az ðɛ ʃap, ov ðɛ kwodraŋgl. Ðɛ kwodraŋgl wil bɛ lqrjest hwen ðɛ opꝏzit aŋglz, aded tɯgeðer, qr ɛkwal tɯ tꝏ rịt aŋglz.



In ðis figyqr, ðɛ aŋglz B and D aded tɯgeðer wɯd bɛ mꝏr ðan tꝏ rịt aŋglz. Wɛ kɯd, ðen, mak ðɛ kwodraŋgl lqrjer, wiðɜt ɵltering ðɛ leŋθ ov its sịdz, bị drɵiŋ B and D apqrt, and krɜdiŋ A and O tɯgeðer, until ðɛ tꝏ aŋglz, B and D, aded tɯgeðer, mad tꝏ rịt aŋglz. Ðɛ aŋglz A and O wɯd, ɵlsꝏ, aded tɯgeðer, ðen bɛ ɛkwal tɯ tꝏ rịt aŋglz, and ðɛ kwodraŋgl wɯd bɛ az lqrj az wɛ kɯd mak it wiðɜt çanjiŋ ðɛ leŋθ ov ðɛ sịdz.

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must ị la fꝏr stiks ov unɛkwal leŋθ on ðɛ flꝏr sꝏ az tɯ enklꝏz ðɛ lqrjest spas. ʕHɜ must ị plas ðem hwen ðɛ stiks kan bɛ divịded intɯ tꝏ kupls ov ɛkwal leŋθ. Let ðɛ teçer ʃꝏ ðɛ çịld ðat ðạr qr *tꝏ* diferent ɑnserz tɯ ðɛ lɑst kwestyon; wun ɑ paralelꝏgram, ðɛ uðer not. And, giviŋ ɑ çịld fꝏr unɛkwal stiks, let him trị hɜ meni kwodraŋglz, ɵl ov ðɛ lqrjest posibl sịz, hɛ kan mak.

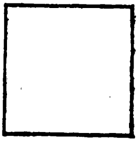
7. Hwen ɑ paralelɷgram iz put intɯ its lɋrjest fɵrm, az đə opɷzit aŋglz ɋr ɵlwaz ɛkwal, and đɛ tɷ ɋr ɛkwal tɯ tɷ rįt aŋglz, ɛç ov đə fɷr aŋglz wil be ɑ rįt aŋgl.

ÇAPTER XIV.

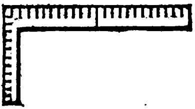
REKTAŊGLZ AND SKWARZ.

1. Hwen ɛç aŋgl ov ɑ paralelɷgram iz ɑ rįt aŋgl, đɛ figyųr iz kɵld ɑ rɵktaŋgl. Đis iz ɑ veri komon figyųr in ɵl sɵrts ov ŧiŋz đat men mak. Đə panz ov glɑs in đə windɷz, and đɛ windɷz đemselvz, đɛ dɷrz and đɛ panelz in đem, đɛ sįdz ov đə rɷm, đə lɛvz ov bɯks, ʃets ov paper, and meni uđer ŧiŋz, ɋr yųʒųali mad in đɛ ʃap ov rektaŋglz. But in đɛ ŧiŋz đat wȩr not mad bį men đąr ɋr veri fų rektaŋglz; đa ɋr skąrsli tɯ be fɜnd ɛvn in kristalz; kɷrs sɵlt and įurn pirįtez beiŋ đə ɷnli komon ŧiŋz in hwiç đə Kreator haz yųzd đə rektaŋgl.

¿Hwot is đɛ jeɷmetrikal nam ov đə figyųr ov ɑ pan ov glɑs. Nam ɵl đɛ objekts yɷ kan ŧiŋk ov hwiç ɋr rektaŋgųlɑr in ʃap. ¿Hwot natyųral objekts prɛzent đɛ fɵrm ov ɑ rektaŋgl. ¿If



2. Hwen əl đɛ sịdz ov ɑ rektaŋgl ɋr ɛkwal, đɛ figyɥr iz kəld ɑ skwɋr. And, az đɛ kərnərz ov ɑ skwɋr ɋr əl rịt aŋglz, sɷ ɑ rịt aŋgl iz sumtịmz kəld ɑ skwɋr kərner.



3. Hwen kɋrpenterz wiʃ tɯ mɋrk ɑ rịt aŋgl on đɋr bɷrdz ər timber, đa yɷz ɑ simpl tɷl, hwiç ị dɋr sa yɷ hav sɛn, and hwiç iz kəld a kɋrpenterz skwɋr.

4. But, in Jɛometri, đɛ wurd skwɋr iz ɷnli yɷzd tɯ men ɑ rektaŋgl wiđ ɛkwal sịdz.

5. Ɨ supɷz yɷ nɷ đɛ wa in hwiç men meʒɥr hʊ loŋ ɑ ŧiŋ iz, wiđ ɑ rɷl, ər yɋrd-stik, ər tap. Đa meʒɥr hʊ meni inçez, ər fɛt, ər yɋrdz, ər mịlz, it wil tak tɯ streç aloŋ bị đɛ sịd ov đɛ ŧiŋ đa ɋr meʒɥriŋ.

6. Men meʒɥr surfasez, suç az pantiŋ, ər kɋrpetiŋ, ər fɛldz, bị fịndiŋ ʊt hʊ meni skwɋrz, hɷz sịdz ɋr ɛç wun inç, ər wun fut, ər wun yɋrd, it wil tak tɯ kuver đɛ surfas hwiç đa wont tɯ meʒɥr. Hwen ɑ kɋrpet-deler selz sɷ menị yɋrdz ov əl-kloŧ ər əl-kɋrpetiŋ, hɛ menz đat đɛ kɋrpet kɯd bɛ kut in suç ɑ wa az tɯ mak just đat number ov skwɋrz, wun yɋrd

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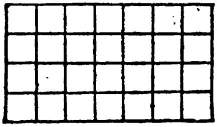
yɷ fɜnd ɑ rektaŋgɥlɑr pɛs ov stɷn ər jurn, wɯd yɷ ɛzili bɛlɛv đat men had not kut it intɯ đat ʃap. ¶Hwot iz ɑ kɋrpenterz skwɋr. ¶Hwot iz ɑ skwɋr kərner. ¶Hwot iz đɛ jɛon ɑ sįd. Hwen ɑ man sez đat đąr ɑ̨r sɷ meni fɛt ov land in hiz dɷr-yɑ̨rd, hɛ mɛnz đat it wɯd tak just đat number ov skwąr pɛsez ov paper, wun fɯt on ɑ sįd, tɯ kuver hiz yɑ̨rd.

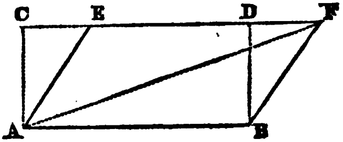
7. It iz veri ɛzi fɵr eni wun đat nɷz đɛ multiplikaʃon tabl, tɯ meʒųr ɑ rektaŋgl. Fɵr đɛ number ov skwąr inçez, ɵr yɑ̨rdz, in ɑ rektaŋgl iz fȣnd bį multiplįiŋ đɛ number ov inçez ɵr yɑ̨rdz, in đɛ bredŧ bį đɛ number in đɛ widŧ, and đis wil giv đɛ number ov skwąrz in đɛ rektaŋgl.



If yɷ hav lęrnd tɯ kȣnt, yɷ wil sɛ bį đis figyųr đat ɑ rektaŋgl fɷr fɛt wįd and sevn fɛt loŋ, kɯd bɛ divįded intɯ twenti-at skwąrz, ɛç bɛiŋ ɑ fɯt on ɑ sįd. Wɛ kan divįd it intɯ sevn rɷz, ov fɷr skwąr fɛt in ɑ rɷ, ɵr intɯ fɷr rɷz, sevn skwąr fɛt in ɑ rɷ.

8. Hwen wɛ wiʃ tɯ meʒųr ɑ paralelɷgram đat iz not ɑ rektaŋgl, wɛ hav ɷnli tɯ multiplį đɛ leŋŧ bį đɛ bredŧ; bekɵz everi paralelɷgram iz egzaktli đɛ sam sįz az ɑ rektaŋgl ov đɛ sam leŋŧ and bredŧ wɯd bɛ.

ɷmetrikal mɛniŋ ov đɛ wurd skwąr. ʕHȣ dɷ men meʒųr đɛ leŋŧ ov ŧiŋz. ʕHwot dɷ đa men bį saiŋ ɑ hundred fɛt ov land. ʕHȣ dɷ đa fįnd ȣt đɛ number ov skwąr fɛt ɵr skwąr inçez in ɑ rektaŋgl. ʕIn ɑ paralelɷgram. Let đɛ tɛçer kopi đɛ last figyųr on đɛ blakbɷrd, and ɑsk đɛ çildren hȣ, if đɛ rektaŋgl and paralelɷgram węr mad ov paper, đa kɯd kut đɛ rektaŋgl



Yꝏ sɛ in đɛ figyųr đat đɛ rektaŋgl A B C D iz ov đɛ sam leŋŧ and bredŧ az đɛ paralelꝏgram A B F E. And đa qr ov egzaktli đɛ sam sįz. If yꝏ kut đɛ trįaŋgl C A E of wun end ov đɛ rektaŋgl and plas it at đɛ uđer end, it wil just kuver đɛ trįaŋgl B D F.

9. Hwen į sa đat wɛ multiplį đɛ leŋŧ ov ɑ paralelꝏgram bį its bredŧ in ərder to fįnd its meʒųr, į mɛn, ov kꝏrs, đat wɛ must multiplį đɛ *number* ov inçez in đɛ leŋŧ bį đɛ *number* ov inçez in đɛ bredŧ in ərder tɯ fįnd đɛ *number* ov skwąr inçez, đat iz, skwąrz an inç on ɑ sįd, đat it wɯd tak tɯ kuver đɛ paralelꝏgram. *Numberz* qr đɛ ꝏnli ŧiŋz đat kan bɛ multiplįd.

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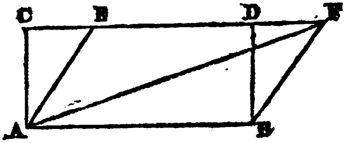
tɯ mak it kuver đɛ paralelꝏgram, and hɤ đɛ paralelꝏgram tɯ mak it kuver đɛ rektangl.

ÇAPTER XV.

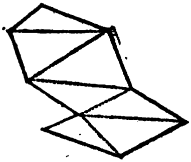
TRĮAŊGLZ AND REKTAŊGLZ.

1. Everi trįaŋgl ma bɛ imajind az hqf ov ɑ paralelꝏgram. If wɛ had ɑ trįaŋgl A B F, wɛ kɯd drə A E paralel tɯ B F, and F E paralel tɯ B A, and đat wɯd mak ɑ paralelꝏgram just twįs az lqrj az đɛ trįaŋgl.



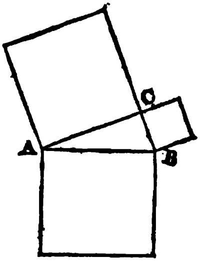
2. And đus wɛ ma fįnd đɛ meʒųr ov ɑ trįaŋgl; đat iz, wɛ ma fįnd hʊ meni skwąrz ov wun inç on ɑ sįd, ər ov wun fɯt on ɑ sįd, it wɯd tak tɯ kuver đɛ trįaŋgl egzaktli. Wɛ nɛd ǫnli multiplį eni wun sįd ov đɛ trįaŋgl bį đɛ distans tɯ ɑ paralel lįn drən ꝟrꝏ đɛ opꝏzit vęrteks. Đis wil giv us đɛ meʒųr ov ɑ paralelꝏgram, and hqf ov đat wil giv us đɛ sįz ov đɛ trįaŋgl.

3. In đis wa men meʒųr surfasez ov everi ʃap, bį divįdiŋ đem intɯ trįaŋglz, and đen fįndiŋ ʊt hʊ lqrj ɛç trįaŋgl iz. Yꝏ ma drə upon yꝏr slat figÿųrz ov eni number ov strat sįdz, and đen divįd đem intɯ trįanglz bį drəiŋ dįagꝏnalz. Yꝏ kan divįd đɛ sam



figyųr intɯ diferent sets ov trįaŋglz bį drəiŋ diferent dįagꝏnalz.

4. In everi rįt trįaŋgl đɛ skwąr mad on đɛ sįd opꝏzit đɛ rįt aŋgl iz just az lɋrj az đɛ skwąrz on đɛ uđer tꝏ sįdz, pɯt tɯgeđer. If, in mį figyųr, đɛ trįaŋgl A B C haz ɑ rįt aŋgl at C, đɛ skwąr on đɛ sįd A B wil bɛ just az lɋrj az đɛ skwąr on A C aded tɯ đɛ skwąr on C B.



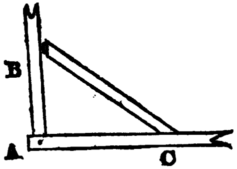
5. Đɛ sįd ov ɑ rįt trįaŋgl opꝏzit đɛ rįt aŋgl iz kəld đɛ hįpotɛnųs, and đɛ uđer sįdz ɋr kəld đɛ legz. Sꝏ đat hwot į hav əlredi tꝏld yꝏ ma bɛ rɛpɛted in diferent wurdz: Đɛ skwąr on đɛ hįpotɛnųs iz ɛkwivalent tɯ đɛ sum ov đɛ skwąrz on đɛ legz. Đis iz kəld đɛ Piŧagꝏrɛ́an propꝏziʃon, bɛkəz it woz fȣnd ȣt bį ɑ jɛometer, hꝏ livd mꝏr đan tꝏ ŧȣzand yerz agꝏ, hꝏz nam woz Piŧagꝏras.

6. Đis Piŧagꝏrɛ́an propꝏziʃon givz us ɑ gɯd wa ov trįiŋ hweđer an aŋgl iz ɑ rįt aŋgl. If đɛ sum ov đɛ skwąrz on tꝏ sįdz ov ɑ trįaŋgl iz just ɛkwal tɯ đɛ skwąr on đɛ ŧẹrd sįd, wɛ ma nꝏ đat đɛ aŋgl opꝏzit đis ŧẹrd sįd iz ɑ rįt aŋgl.

¿Hȣ dꝏ men meʒųr trįaŋglz. ¿Hȣ dꝏ đa meʒųr uđer surfasez đan paralelꝏgramz. ¿Hȣ loŋ agꝏ did Piŧagꝏras liv. ¿Hwot did hɛ diskuver abȣt rįt trįaŋglz. ¿Hȣ kan yꝏ tel

Nʊ, ɑ skwąr iz ɑ rektaŋgl az bred az it iz loŋ; sω đat, tɯ fįnd hʊ lqrj đɛ skwąr on ɑ leg wɯd bɛ, wɛ ned ωnli multiplį đɛ number ov inçez in đɛ leŋŧ ov đɛ leg bį itself. If yꝏ ʃɯd meʒųr đɛ sįdz ov ɑ trįaŋgl, and fįnd đat wun sįd woz ŧrɛ inçez, and anuđer fωr inçez, and đɛ ŧẹrd fįv inçez; đen, if yꝏ nω đɛ multiplikaʃon tabl, yꝏ wɯd nω đat đɛ skwąrz on đɛ sįdz wɯd bɛ nįn inçez, and sikstɛn inçez, and twenti-fįv inçez. Mωrωver, if yꝏ ad nįn tɯ sikstɛn, it maks twenti-fįv. Sω đat in ɑ trįaŋgl hꝏz sįdz qr ŧrɛ, fωr and fįv inçez, đɛ sįd ov fįv inçez iz ɑ hįpotenųs, and đɛ opωzit aŋgl ɑ rįt aŋgl.

7. If yꝏ understand đɛ last sekʃon, yꝏ wil elsω understand hʊ đɛ Piŧagωrean propωziʃon, đat, đɛ skwąr on đɛ hįpotenųs iz ɛkwivalent tɯ đɛ sum ov đɛ skwąrz on đɛ legz, iz veri yųsfɯl tɯ kqrpenterz and uđer pẹrsonz hꝏ wiʃ tɯ mak skwąr wurk. Supωz yꝏ wẹr makiŋ ɑ fram ov ɑ hʊs, and wonted đɛ timberz A B and A C tɯ mak ɑ rįt aŋgl at A. Yꝏ mįt meʒųr ŧrɛ fɛt from A tɯ B, and fωr fɛt from A tɯ C, and đen elter đɛ aŋgl at A, bį mꝏviŋ wun ov đɛ timberz until ɑ stik fįv fɛt loŋ wɯd just reç from C



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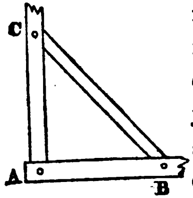
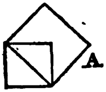
hweđer ɑ trįaŋgl iz ɑ rįt trįaŋgl. ʕHʊ dꝏ kqrpenterz mak đɛ kerner ov ɑ fram skwąr. ʕHʊ dꝏ đa kut brasez fer skwąr tɥ B. Insted ov ŧrɛ, fωr and fịv, fer lɋrj framz, ɗa tak siks, at and ten fɛt.

8. Anuɗer yɥs ɗat kɋrpenterz mak ov ɗɛ Piŧagω-rɛan propωziʃon iz, in kutiŋ brasez fer framz. Ɗa

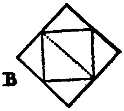


mezɥr ɗɛ sam number ov fɛt from A tɥ B and from A tɥ C; and ɗen fer ɗɛ bras, B C, ɗa mezɥr just az meni tịmz sevntɛn inçez az ɗạr ɋr fɛt in A B. Ɗis maks ɗɛ bras ɑ veri litl tω loŋ; bekɛz ɑ skwạr on sevntɛn inçez iz ɑ veri litl mωr ɗan twịs az lɋrj az ɑ skwạr fut. But kɋrpenterz lịk ɗɛ bras tɥ be ɑ veri litl tω loŋ, bekɛz hameriŋ ɗɛ timberz tɥgeɗer maks ɗɛ bras indent ɗɛ timberz ɑ litl; and, if ɗɛ bras wẹr not ɑ veri litl tω loŋ at fẹrst, it wɥd be ɑ litl tω ʃort ɑfter it had indented ɗɛ timberz; and ɗat wɥd mak A les ɗan ɑ rịt aŋgl.



9. Ɗɛ skwạr on ɗɛ dịagωnal ov ɑ skwạr iz just twịs az lɋrj az ɗat skwạr. Ɗis yω wil sɛ in ɗɛ figyɥr mɋrkt A.

Ɑ skwạr wiɗ its kernerz in ɗɛ midl ov ɗɛ sịdz ov anuɗer skwạr iz just hɋf az lɋrj az ɗat skwạr. Ɗis yω wil sɛ in ɗɛ figyɥr mɋrkt B.



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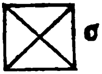
kerners. ꞋHɤ lɋrj is ɑ skwạr bilt on ɗɛ dịagonal ov ɑ skwạr. ꞋHɤ lɋrj is ɑ skwạr hωs kernerz ɋr in ɗɛ midl ov ɗɛ sịds ov

If wε fansi ɑ skwąr kut intɯ fɷr rįt trįaŋglz bį tꝏ dįagɷnalz (az at c,) wε kan fansi εç litl



trįaŋgl turnd ɷver on its ɷn litl hįpotenųs, l wiç wil mak ɗε figyųr at ʙ ɑ skwąr just twįs az lɋrj az ɗat at c.

Yꝏ ma tak ɑ skwąr pes ov paper, az at ʙ, fɷld εç kεrner ɷver tɯ ɗε senter ov ɗε skwąr, az at c, ɗen unfɷld it agen, az at ʙ, and į ŧiŋk ɗis wil mak yꝏ understand ɗis sekʃon.

ÇAPTER XVI.

SERKLZ.

1. Wε hav had fiftεn çapterz abȣt strat lįnz, and abȣt figyųrz bȣnded bį strat lįnz. Ɨ ŧiŋk, ɗąrfɷr, ɗat yꝏ wil bε glad tɯ lẹrn ɑ litl abȣt kurvz.

2. Ɑ kurv iz, az į hav tɷld yꝏ, ɑ lįn ɗat bendz in everi pɋrt ov it, but haz nɷ ʃɋrp kεrnerz. If ɑ bȣ

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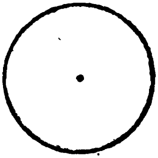
anuɗer skwąr. Let ɗε çildren dra rįt trįaŋgls and skwąrz upon ɗε sįdz, and tel ɗε relativ sįz ov ɗε skwąrz akerdiŋ tɯ ɗε Piŧagɷrεan propɷziʃon. Ɗε teçer ʃud rεkwįr ɗis propɷziʃon, az givn ɑfter ɗε kɷlon on paj 45, tɯ bε komited tɯ memɷri, az it iz ɗε mɷst yųsfɯl propɷziʃon in ɗε fiftεn çapterz.

ʕ Hwot iz ɑ kurvd lįn. ʕ Hwot iz ɑ strat lįn. ʕ Hȣ kꝏd yꝏ mɋrk ɑ strat lįn wiɗ ɗε hwεl ov ɑ hwεlbarɷ. ʕ Hȣ wꝏd yꝏ

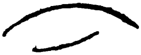
wȩr hwɛliŋ ɑ bɑrω in ɑ ląrj, level fɛld, just ɑfter ɑ lįt snω, ɗɛ midl ov ɗɛ trɑk ov hiz hwɛl wɯd bɛ ɑ lįn ɗɑt yω kɯd ɛzili folω wiɗ yωr į.

If ɗɛ bɷ went ɵl ɗɛ tįm in ɗɛ sam direkʃon, hɛ wɯd mak ɑ strat lįn; but if hɛ kept turniŋ ɑ litl ɵl ɗɛ hwįl tɯ ɗɛ rįt ɵr left, hɛ wɯd mak ɑ kurv.

3. If ɗɛ bɷ kept on ɵl ɗɛ tįm turniŋ in ɗɛ sam direkʃon, just az fɑst az hɛ began tɯ turn, ɗɛ trɑk ov ɗɛ hwɛl wɯd kum rʊnd intɯ itself, and mak hwot iz kɵld ɗɛ sȩrkumferens ov ɑ sȩrkl. Ɑ kurv ɗɑt bendz ɛkwali in everi pąrt, iz kɵld ɑ sȩrkumferens; and ɑ figyųr bʊnded bį ɑ sȩrkumferens iz kɵld ɑ sȩrkl.



4. Pąrts ov ɑ sȩrkumferens ąr kɵld ąrks. Ɗɛ wurd ąrk yųzd tɯ mɛn ɑ bω; and bωz ąr ʃapt sumƭiŋ lįk an ąrk.



5. If ɗąr woz ɑ trɛ in ɗɛ midl ov ɑ fɛld, and ɗɛ bɷ ʃɯd kɛp ɵl ɗɛ tįm at ɗɛ sam distans from ɗɛ trɛ, hɛ wɯd kum rʊnd tɯ ɗɛ plas hɛ stąrted from, and ɗɛ trɑk ov hiz hwɛl wɯd bend ɛkwali in everi pąrt.

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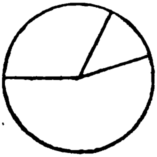
mąrk ʊt ɑ kurv wiɗ ɗɛ hwɛl. If ɑ lįn bendz ɛkwali in everi pąrt, ¿hʊ loŋ kan it bɛ. (Ωnli loŋ ɛnuf tɯ return intɯ its ωn beginiŋ.) ¿Hwot iz ɑ kurv kɵld ɗɑt haz nω endz, and bendz ɛkwali in everi pąrt. But, if it bendz ɛkwali in everi

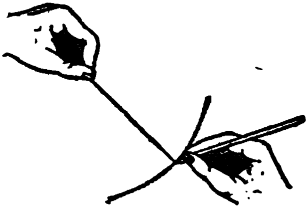
Hɛ wɯd, in fakt, gꝏ rɜnd in đɛ sęrkumferens ov ɑ sęrkl.

6. Yꝏ wil sɛ, in đɛ figyɑ̨r ov ɑ sęrkl in sekʃon ŧrɛ, ɑ dot in đɛ midl. It ʃꝏz đɛ plas hwɑ̨r wɛ supꝏz đɛ trɛ tɯ stand. And in ɵveri sęrkl đɑ̨r iz ɑ pœnt in đɛ midl, ɛkwali distant from everi pɑ̨rt ov đɛ sęrkumferens. Đis pœnt iz kɵld đɛ senter ov đɛ sęrkl. It iz not ɵlwaz mɑ̨rkt bį ɑ dot, but đɛ plas iz ɵlwaz đɑ̨r hweđer it iz mɑ̨rkt ɵr not.



7. Ɑ strat lįn drɵn from đɛ senter ov ɑ sęrkl tɯ its sęrkumferens iz kɵld ɑ radius. Θl đɛ radiį ov đɛ sam sęrkl ɑ̨r ɛkwal. In đis figyur đɑ̨r ɑ̨r ŧrɛ radiį mɑ̨rkt bį blak lįnz.

8. Wɛ kan fansi ɑ lįn mad bį ɑ radius swiŋiŋ ɵl rɜnd ɑ senter. And yꝏ ma drɵ veri nįs sęrklz bį hɷldiŋ, wiđ wun hand, ɑ ŧred fast tɯ wun spot on yꝏr slat, ɵr blakbꝏrd, hwįl, wiđ yꝏr uđer hand, yꝏ hɷld đɛ uđer end ov đɛ striŋ and yꝏr pensil tugeđer and drɵ ɑ



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pɑ̨rt, and yet has tꝏ endz, ¿hwot is it kɵld. Hwen į swiŋ đɛ dꝏr on its hinjez, ¿in hwot kurv duz đɛ end ov đɛ laç mꝏv. ¿Hwɑ̨r is đɛ senter ov đɛ ɑ̨rk. Hwen ɑ stꝏn haŋz bį ɑ ŧred and swiŋz bakward and fɵrward ꝏver ɑ strat lįn on đɛ flꝏr sẹrkumferens, kepiŋ đe striŋ streçt. Yω kan ɵlsω drɵ đem wiđ wun hand, bį restiŋ yωr litl fiŋger on đe senter ov đɛ sẹrkl, and beiŋ kąrful tu kep đɛ end ov yωr kraon ɵlwaz at đɛ sam distans from it. Đis wa ωnli ɑnserz fɵr đe blakbωrd.

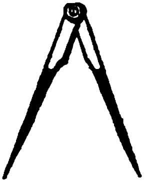
ȻAPTER XVII.

MΩR ABɤT SERKLZ.

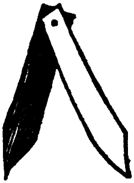
1. Rektaŋglz and sẹrklz ąr đɛ mωst komon figyųrz in ɵl manyųfaktyųrd ŧiŋz. Đa ąr đe ɛziest figyųrz tu mak egzakt, and đɛ mωst konvenyent hwen mad. Yω hav ɵlredi nωtist hɤ komon rektaŋglz ąr in hwot men mak. Sẹrklz ąr ɵlmωst az komon. Butnz and dωr-nobz, plats, panz, and hwɛlz ąr sẹrkųlɑr. But in natyųral ŧiŋz—in ŧiŋz mad bį đɛ grat Krɛator—sẹrklz ąr muç mωr komon đan rektaŋglz. Đɛ sun, đɛ mωn and stąrz, đɛ budz ov meñi flɤerz, đɛ įz ov animalz, and sum litl animalz đemselvz, ąr nɛrli in đɛ ʃap ov ɑ sẹrkl.

ʕhwot kurv duz đɛ stωn mωv in. ʕHwąr iz đɛ senter ov đɛ ąrk. ʕHwot lįn duz đɛ ŧred mąrk ɤt. Let đɛ teçer aktyųali ωpn and ʃut đɛ dωr, and swiŋ ɑ plumb, hwįl ɑskiŋ đɛz kwestyonz. Kep đɛ çįld konstantli in đɛ habit of sɛiŋ đɛ pẹrfekt jɛωmetrikal fɵrmz sugjested bį đɛ material aperansez. Let đem drɵ sẹrklz, and đąr radiį oŋ đɛ bωrd.

ʕHwot iz đɛ figyųr ov ɑ sent. ʕHwot uđer ŧiŋz kan yω tel mɛ ov in đɛ ʃap ov ɑ sẹrkl. ʕIz đɛ sẹrkl ɵr đɛ rektaŋgl



2. Tɯ drɵ sȩrklz, men hav hwot iz kɵld ɑ pąr ov kumpasez, đat ωpn and ʃut lįk ɑ pąr ov toŋz. Đa pɯt wun pɵnt dȣn on đe paper hąrd ɛnuf tɯ hωld it stil in đe senter ov đe sȩrkl, and đen mωv đe uđer leg rȣnd lįtli, just bąriŋ on hąrd ɛnuf tɯ mąrk đe sȩrkumferens. Đe sȩrkl wil be ląrjer ɵr smɵler, az đe kumpasez ąr ωpnd wįder, ɵr ąr mωr nerli ʃut.



Yω kan mak ɑ pąr ov kumpasez, tɯ drɵ sȩrklz on đe grȣnd, bį pɯtiŋ wun smɵl nal ŧrω tω bits ov ʃiŋgl, and hwitliŋ đe endz tɯ ɑ pɵnt. Đe ωld Grɛks ov Piŧágωras tįm, and ɑfterwɑrdz, yųzd tɯ studi and teç Jɛometri wiđ đe help ov figyųrz drɵn on smωđli spred sand.

3. Tɯ mak ŧiŋz sȩrkųlɑr-ʃap, men sumtįmz drɵ ɑ sȩrkl fȩrst, wiđ kumpasez, and đen kut đe ŧiŋ tɯ đat ʃap. But jenerali đa yųz ɑ diferent meŧod. Đa yųz ɑ turniŋ-lađ, ɵr sumŧiŋ đat wurks on đe sam plan. In ɑ turniŋ-lađ, ɑ blok ov wɯd ɵr pes ov įurn iz mad tɯ turn stedili rȣnd on ɑ stɛl pɵnt đat mąrks đe senter, hwįl đe çizel, đat kuts đe wɯd ɵr įurn, iz

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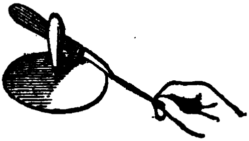
ofner fȣnd in natyųral objekts. ⸮Hwot instrωment dω men hav tɯ drɵ sȩrklz wiđ. ⸮Hȣ kan yω mak ɑ sɵrt ov kumpasez. ⸮Hwot did đe ωld Grɛks yųz fɵr ɑ blakbωrd and çɵk. ⸮Did yω ever sɛ ɑ turniŋ-lađ. ⸮Ɑ potɛrz hwɛl. ⸮Ɑ tiner at held stedili at đe sam distans from đe stel pɷnt, sɷ đat đe wɯd er jurn đat iz tɷ fɋr from đe senter iz kut of az it gɷz past đe çizel. On ɑ poterz hwel, ɑ lump ov kla iz mad tɯ turn rɜnd, hwjl its senter remanz in wun plas; and đus đe poter maks đe krokeri rɜnd. Đe botom ov tind panz iz kut ɜt bj ɑ sert ov sizorz đat iz fasnd at đe rjt distans from ɑ pɷnt rɜnd hwiç đe pes ov tind plat iz mad tɯ turn.



Sumtjmz bɷz mak ɑ sȩrkl ov leđer fer ɑ plaŧiŋ. Put an el ŧrɷ ɑ bit ov leđer intɯ ɑ bɷrd; đen stik ɑ ʃɋrp njf ŧrɷ đe leđer intɯ đe bɷrd, slantiŋ, wiđ đe kutiŋ ej dɜn; đe blad at rjt aŋglz tɯ ɑ ljn jɷniŋ it tɯ đe el. Nɜ pɯl đe leđer rɜnd đe el, under đe njf, and yɷ wil kut ɜt ɑ sȩrkl. Put ɑ stroŋ striŋ ŧrɷ đe el-hɷl in đe leđer, wiđ ɑ lɋrj not at đe end ov it, and yɷ wil hav ɑ kɋrius tɷ. Wet đe leđer and pres it under yɷr fɯt upon ɑ flat stɷn, and yɷ kan lift đe stɷn bj đe striŋ. But be kɋrfɯl not tɯ swiŋ đe stɷn abɜt; fer nɷbodi haz ɑ rjt tɯ pɯt uđer pepl in danjer.

Θl đez waz ov makiŋ ɑ sȩrkl ɋr sumhwot aljk. But đɋr iz anuđer wa—tɯ bend wjr, er ŧiŋz ov đe

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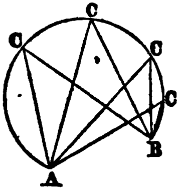
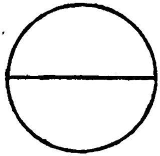
wurk kutiŋ ɜt rɜnd pesez. ¿Hɜ did he kut đem. ¿Did yɷ ever se ɑ blaksmiŧ bend ɑ wagon tjr. ¿Hwj duz pasiŋ đe tjr ŧrɷ đe ŧre rɷlerz mak ɑ sȩrkl ov it. ¿Hɜ duz ɑ kɷper mak kɉnd, ɛkwali at everi pɑrt. Đus, đɛ blaksmiŧ pasez an ɉurn bɑr betwɛn ŧrɛ rωlerz, đat bend ɛç pɑrt ov đɛ tɉr ɛkwali, and đus briŋ it intɯ ɑ sẹrkụlar fẹrm fer ɑ wagon-tɉr.

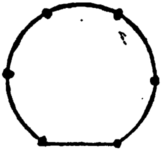


4. Ɑ strat lɉn jωniŋ đɛ endz ov an ɑrk, iz keld ɑ kerd. Eni strat lɉn gωiŋ akros ɑ sẹrkl, haviŋ bωŧ endz in đɛ sẹrkumferens, iz ɑ kerd. Θl đɛ lɉnz mɑrkt A C and B C, in đis figyụr ɑr kerdz.



5. Đɛ loŋgest kerd đat wɛ kan hav, iz đɛ wun đat gωz ŧrω đɛ senter ov đɛ sẹrkl. It iz keld đɛ dɉameter, and iz just twɉs đɛ leŋŧ ov ɑ radius.

6. If wɛ divɉd ɑ sẹrkumferens intɯ siks ɛkwal ɑrks, đɛ kerd ov ɛç ɑrk iz just az loŋ az ɑ radius.



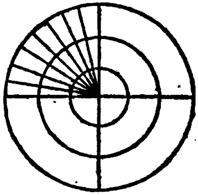
Sω đat, if yω drɛ ɑ sẹrkl on đɛ grɜnd wiđ yωr ʃiŋgl kumpasez, yω wil fɉnd if yω ɑr kɑ̣rful neđer tɯ ωpn ner klωz yωr kumpasez, đat đá wil step rɜnd đɛ sẹrkumferens in egzaktli siks steps.

ɑ hωp rɜnd. Let đɛ teçer drɛ ɑ sẹrkl wiđ radius, kerd, and dɉameter, and ɑsk hwot đɛ nɛm ov ɛç lɉn iz. ʕHwot iz đɛ loŋgest kerd wɛ kan hav. If wɛ divɉd đɛ hωl sẹrkumferens intɯ siks ɛkwal ɑrks, ʕhɜ loŋ iz đɛ kerd ov ɛç ɑrk. ʕHɜ kan yω ʃω it wiđ ɑ pɑr ov kumpasez.

ȻAPTER XVIII.

MEƷURIŊ AŊGLZ.

1. If đɛ vęrteks ov ɑ rįt aŋgl iz at đɛ senter ov ɑ sęrkl, đɛ lįnz fɵrmiŋ đɛ aŋgl kut of wun kwɵrter ov đɛ sęrkumferens, hweđer đɛ sęrkl iz lɑrj ɵr smɵl. Sω wiđ eni aŋgl hωz vęrteks iz at đɛ senter ov ɑ sęrkl; its sįdz ɵlwaz kut of just đat pɑrt ov ɑ sęrkumferens hwiç đɛ aŋgl iz ov fωr rįt aŋglz. If it taks at ov đɛz litl aŋglz tɯ mak wun rįt aŋgl, it wil tak at ov đɛ litl ɑrks tɯ mak ɑ kwɵrter ov ɑ sęrkumferens, hweđer on ɑ lɑrj ɵr smɵl sęrkl.



If wɛ wont tɯ tel hɤ lɑrj an aŋgl iz, wɛ tel hwot pɑrt ov ɑ sęrkumferens it wɯd kut of if its vęrteks węr at đɛ senter.

2. And, in ɵrder đat wɛ ma ɛzili tel đɛ pɑrts ov ɑ sęrkumferens, wɛ fansi everi sęrkumferens az divįded intɯ kwɵrterz, and đen ɛç ov đɛz kwɵrterz divįd-

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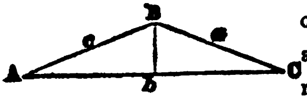
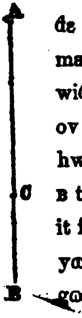
ʕHɤ meni dɛgrɛz ov ɑrk mak ɑ kwɵrter ov ɑ lɑrj sęrkumferens. ʕHɤ meni dɛgrɛz mak ɑ kwɵrter ov ɑ smɵl sęrkumferens. If yω pɯt đɛ vęrteks ov ɑ rįt aŋgl at đɛ senter ov ɑ sęrkl, ed intɯ nịnti ekwal pq̣rts, hwiç q̣r keld degrez ov q̣rk. Sɷ ɗat we tel hʊ lq̣rj an aŋgl iz bị teliŋ hʊ meni ov ɗez degrez wɯd be kut of betwen its sịdz if its vẹrteks wẹr at ɗe senter ov ɑ sẹrkl.

3. If ɗe vẹrteks ov ɑ rịt aŋgl iz plast at ɗe senter ov ɑ sẹrkl, its sịdz kut of ɑ hɷl kwerter sẹrkumferens, and ɗe rịt aŋgl iz ɗq̣rfɷr sumtịmz keld an aŋgl ov nịnti degrez. Hq̣f ɑ rịt aŋgl, lịk ɗe aŋglz ov an ịsoselez rịt trịaŋgl, iz keld an aŋgl ov ferti-fịv degrez.



4. Az eç rịt aŋgl iz an aŋgl ov nịnti degrez, tɷ rịt aŋglz tɯgeɗer wil mak an aŋgl ov twịs nịnti, ɗat iz, ov wun hundred and àti degrez.



5. Yɷ ma tak eni pɷnt on ɑ strat lịn, az ɗe pɷnt c in ɗe lịn A B, and fansi ɗe lịn az makiŋ an aŋgl ov wun hundred and ati degrez wiɗ itself at ɗat pɷnt. An aŋgl iz ɗe diferens ov direkʃon ov tɷ lịnz. Nʊ yɷ kan not tel hweɗer ị mɷvd mị pen from A tɯ B, er from B tɯ A; and sɷ, if yɷ lịk, yɷ ma fansi ị mɷvd it from A tɯ C, and ɗen from B tɯ C; ɗat iz, yɷ ma fansi ɗe lịn metiŋ itself at C; ɗat iz, gɷiŋ in opɷzit direkʃonz on eç sịd ov C; ɗat

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Ƕʊ meni degres ov q̣rk wil its sịds kut of. If an aŋgl is an aŋgl ov ferti-fịv degres, Ƕwot pq̣rt ov ɑ rịt aŋgl is it. ƕHav iz, makiŋ an aŋgl ov tꝏ rįt aŋglz at c; đat iz, đɛ lįn maks an aŋgl ov wun hundred and ati degrɛz wiđ itself at c.

6. Yꝏ remember, į hꝏp, đat đɛ ŧrɛ aŋglz ov ɑ trįaŋgl takn tɯgeđer ɋr ɛkwivalent tɯ tꝏ rįt aŋglz. Wɛ kan nȣ sa đɛ sam ŧiŋ in uđer wurdz; wɛ ma sa đat đɛ sum ov đɛ ŧrɛ aŋglz ov ɑ trįaŋgl iz wun hundred and ati degrɛz.

7. In an ɛkwilateral trįaŋgl, yꝏ remember đat đɛ ŧrɛ aŋglz ɋr ɛkwal tɯ ɛç uđer. Ɛç ov đem iz đen an aŋgl ov siksti degrɛz, bekez ŧrɛ tįmz siksti iz wun hundred and ati.

ꞒAPTER XIX.

KƐRDZ.

1. If, insted ov pɯtiŋ đɛ vẹrteks ov an aŋgl at đɛ senter ov ɑ sẹrkl, wɛ put it in đɛ sẹrkumferens, it wil tak in an ɋrk just twįs az lɋrj az it wɯd wiđ its vẹrteks in đɛ senter.

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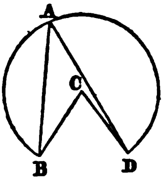
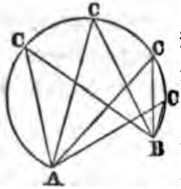
yꝏ ever studid Ariŧmɛtik. ʕDꝏ yꝏ nꝏ hȣ muç nįn tįmz ten mak. ʕĦrɛ tįmz ŧẹrti. ʕHȣ meni aŋglz ov 10° must bɛ pɯt tɯgeđer tɯ mak ɑ rįt aŋgl. ʕHȣ meni degrɛz in wun ŧẹrd ov ɑ rįt aŋgl. ʕHwot aŋgl duz ɑ strat lįn mak wiđ itself. ʕHȣ meni degrɛz dꝏ đɛ ŧrɛ aŋglz ov ɑ trįaŋgl aded tɯgeđer mak. ʕHȣ meni degrɛz in ɛç aŋgl ov an ɛkwilateral trįaŋgl.



2. If wun aŋgl haz its vȩrteks at đɛ senter ov ɑ sȩrkl, az at c in đɛ figyɥr, and anuđer haz its vȩrteks in đɛ sȩrkumferens, az at A, and if đɛ sįdz ov bɷŧ đez aŋglz gɷ ŧrɷ đɛ sȩrkumferens at đɛ sam plasez, D and B, đen wun aŋgl iz just hqf az lqrj az đɛ uđer. Đɛ aŋgl at A iz ɷnli hqf az lqrj az đat at c.

3. Wɛ kan sa đɛ sam ŧiŋ in uđer wurdz. Tɷ kɵrdz stqrtiŋ from đa sam pɷnt, A, in đɛ sȩrkumferens ov ɑ sȩrkl, mak an aŋgl ov just hqf az meni dɛgrez az đąr qr in đɛ qrk D B betwɛn đąr uđer endz. Đɛ qrk D B iz not drɵn at ɵl; but yɷ kan ɛzili fansi it đąr.

4. Nɤ đɛ qrk D B wud bɛ ov đɛ sam leŋŧ, hwąrever đɛ pɷnt A wȩr plast, and đat wil mak yɷ understand đɛ nekst figyɥr.



5. Ɵl đɛ aŋglz đat kan bɛ drɵn in wun qrk, đat iz, wiđ đąr vȩrtisez in đɛ qrk, and đąr sįdz gɷiŋ ŧrɷ đɛ endz ov đɛ qrk, qr ov đɛ sam sįz. Đɛ aŋglz at c, c, c, c, qr ɵl fɷr ɛkwal tɯ ɛç uđer. Ɛç wun iz

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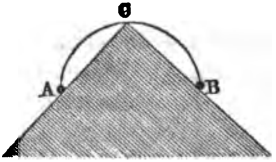
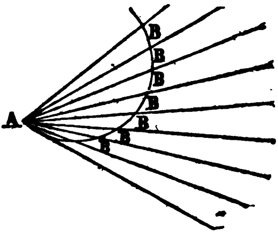
An aŋgl, wiđ its vȩrteks at đɛ senter, iz meʒɥrd bį đɛ qrk betwɛn its sįdz. ʕHɤ iz đɛ aŋgl meʒɥrd hwen its vȩrteks iz in đɛ sȩrkumferens. ʕHɤ iz an aŋgl betwɛn tɷ kɵrdz meʒɥrd, hwen đɛ vȩrteks iz in đɛ sȩrkumferens. ʕKan eni ov đɛ klas meʒɥrd bị hqf ðɛ qrk ðat iz not drən betwɛn **A** and **B**.



6. Ðis givz yɷ ɑ·kɥrius wa ov drəiŋ an qrk ov ɑ sẹrkl. Drịv tɷ pinz in ɑ bɷrd, az at **A** and **B**, and ðen mɷv ɑ bit ov kqrd, wiŧ strat ejez, in suç ɑ wa az ʈɯ kɛp ðɛ kərner ŧrust, az fqr az it wil gɷ, betwɛn ðɛ pinz. Ðɛ kərner **C** wil mɷv in ðɛ qrk ov ɑ sẹrkl.



7. If ɛkwal aŋglz hav ðạr vẹrtisɛz at ðɛ sam pɵnt in ðɛ sẹrkumferens, ða wil kut of ɛkwal qrks. Ðat iz, if ðɛ aŋglz at **A** qr ɵl ɛkwal tɯ ɛç uðer, ðɛ qrks at **B**, **B**, **B**, qr ɛkwal tɯ ɛç uðer.

Ðis iz ðɛ wa in hwiç ralrɷdz qr lȧd ȣt in kurvz. Ðɛ enjinɛr meʒɥrz ɛkwal aŋglz from wun pɵnt, az **A**, and ɛkwal kərdz, az at **B**, **B**, **B**, and ðen ðɛ ralz qr lad tɯ gɷ rȣnd az qrks tɯ ðɷz kərdz.

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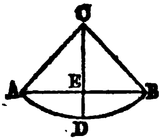
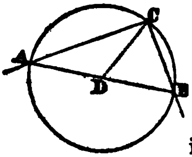
dɵ ɑ figyqr, and eksplan hȣ ɑ kurv in ɑ ralrɷd iz lȧd ȣt. ʕKan yɷ ʃɷ hȣ an qrk ma bɛ drən bị ɑ kqrd and tɷ pinz. Hwen ðɛ kqrd haz ɑ skwạr kərner, ʕhȣ lqrj iz ðɛ qrk. ʕKan eni wun wiŧ ɑ rɷler and ɑ pɛs ov çək ʃɷ ɑ wa ov stakiŋ ȣt ɑ sẹrkqlar gqrden-pqŧ. (Yɥz ðɛ rɷler fər ðɛ stik ov sekʃon atŧ, and çək dots fər staks.) ʕKan yɷ tel mɛ hȣ tɯ trị hweð-

8. Đąr ąr several uđer waz ov dreiŋ sẹrklz wiđɜt kumpasez. Ɨ wil tel yɷ wun mɷr, hwiç yɷ wil fịnd veri yųsful if yɷ ever wont tɯ la ɜt kurvd pąđz in ɑ gąrden, er dɷ eniŧiŋ ov đat kịnd. Tak ɑ strat stik, A E B, and drịv intɯ đe grɜnd tɷ staks, wun at A, and đe uđer at D, makiŋ A D abɜt hąf đe leŋŧ ov đe stik. Kepiŋ wun end ov đe stik at A, mɷv đe uđer end rɜnd until đe midl, E, iz az fąr from D az yɷ ŧiŋk best tɯ put it. Đen drịv ɑ stak at B. Nɜ put wun end ov đe stik at D, and let đe midl, E, be just az fąr from đe stak B as it woz ƀefɷr from D. Drịv ɑ nų stak at đe uđer eŋd ov đe stik. Đus yɷ kan gɷ on, drịviŋ staks az fąr az yɷ wiʃ tɯ gɷ. Đe sịz ov đe sẹrkl wil be mad grater bị makiŋ đe distans D E smeler.



9. If đe ąrk A C B iz just hąf ɑ sẹrkl, đen đe uđer ąrk, A B, iz ɑ hąf-sẹrkl, and đe aŋgl A C B iz meʒųrd bị hąf ɑ hąf-sẹrkl, and iz ɑ rịt aŋgl. If đe kerner ov đe kąrd in sekʃon siks ov đis çapter iz ɑ skwąr kerner, đen đe ąrk wil be ɑ hąf-sẹrkumferens.



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er an aŋgl iz ɑ rịt aŋgl wiđɜt yųziŋ đe Piŧagɷrean propɷziʃon. Invịt đe skolar tɷ dre ɑ figyųr and eksplan. If he kan not, dre it fer him, and ʃɷ, wiđ ʃiŋgl kompasez, if yɷ hav nɷ uđer, hɜ tɯ divịd đe hịpotenųs and aplị đe test ov sekʃon ten.

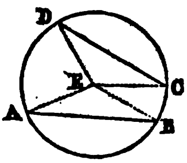
10. If, in ɖe last figyųr, A C B iz ɑ rįt aŋgl, A B wil be ɑ dįameter, and ɖe midl ov it, D, wil be ɖe senter ov ɖe sęrkl, and wil be just az fɑ̧r from C az it iz from A ɵr B.

Ɖis givz us ɑ veri priti wa ov trįiŋ hweɖer an aŋgl iz ɑ rįt aŋgl ɵr not. If į wont tɯ nꝏ hweɖer A C B iz ɑ rįt aŋgl, į wil dre eni lįn A B akros it; and, if hɑ̧f A B wil just reç from ɖe midl ov A B tɯ C, we ma nꝏ ɖat C iz ɑ rįt aŋgl. If it duz not reç, C iz les ɖan ɑ rįt aŋgl; if it mꝏr ɖan reçez, C iz mꝏr ɖan ɑ rįt aŋgl.

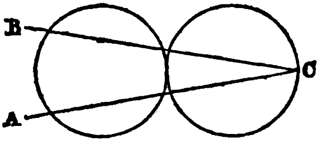
CAPTER XX.

KɵRDZ AND TANJENTS.

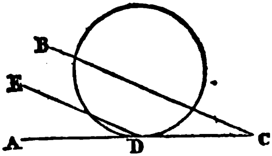
1. Hwen tꝏ kɵrdz dꝏ not tuç eç uɖer, ɖe aŋgl betwen ɖem iz meʒųrd bį hɑ̧f ɖe diferens ov ɖe ɑrks betwen ɖem. Ɖe aŋgl mad bį ɖe kɵrdz A B and C D iz meʒųrd bį hɑ̧f ɖe diferens betwen ɖe ɑrks A D and B C. Ɖat iz tɯ sa, ɖe aŋgl betwen A B and C D iz hɑ̧f az lɑ̧rj az ɖe diferens ov ɖe tꝏ aŋglz at E, A E D and C E B.



2. Wɛ ma imajin ðɛ kerdz leŋŧend intɯ strat lįnz goiŋ ʊtsįd ðɛ sȩrkl untił ða met; and it wil not elter ðɛ sįz ov ðɛ aŋgl. Sɯ ðat ðɛ aŋgl A C B iz meʒųrd bį hɋf ðɛ diferens ov ðɛ ɋrks betwɛn its sįdz, hwąrever wɛ plas ðɛ sȩrkl.



3. If, in ðɛ lɑst figyųr, wɛ mɯv ðɛ sȩrkl bak until ðɛ sȩrkumferens tuçez ðɛ vȩrteks C, ðen ðɛ smeler ɋrk haz bɛkum ov nɯ sįz at el, and ðɛ diferens bɛtwɛn it and ðɛ lɋrj ɋrk iz ɛkwal tɯ ðɛ hɯl ov ðɛ lɋrj ɋrk. Ðen ðɛ aŋgl iz meʒųrd bį hɋf ðɛ lɋrj ɋrk, hwiç iz just ðɛ sam ŧiŋ ðat yɯ lȩrnd in ðɛ lɑst çapter.

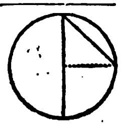


4. If wɛ tak ðɛ sȩrkl fɋrðest from ðɛ vȩrteks in ðɛ lɑst figyųr, and lift it up til it just tuçez ðɛ lįn A C, at ðɛ pɯnt D, az in ðis figyųr, ðen ðɛ lɋrjer ɋrk mɛts ðɛ smeler ɋrk just at D.

Hwen tɯ kerds dɯ not tuç ɛç uðer, ʕ hʊ is ðɛ aŋgl meʒųrd. ʕ Kan yɯ dre a figyųr on ðɛ blakbɯrd, and eksplan ðis mɯr fuli. Hwen tɯ strat lįnz gɯ ŧrɯ a sȩrkl and met ʊtsįd ðɛ sȩrkl, ʕ hʊ iz ðɛ aŋgl betwɛn ðem meʒųrd. ʕ Kan yɯ dre a figyųr, and eksplan ðat. ʕ Hwot iz ment bį a tanjent

5. Đɛ lįn A C iz nʊ kəld ɑ tanjent. Tanjent mɛnz ɑ tuçer; and đɛ lįn A C just tuçez đɛ sȩrkumferens wiđʊt kutiŋ it.

6. If nʊ wɛ mωv đɛ lįn B C wiđʊt çanjiŋ its direkʃon, until it standz in đɛ plas ov E D, đɛ sməl ɋrk haz bekum nuŧiŋ; sω đat đɛ aŋgl A D E iz meʒųrd bį hɋf đɛ ɋrk betwen its sįdz.



7. In uđer wurdz, đɛ aŋgl betwen ɑ kərd and ɑ tanjent at wun end ov đɛ kərd iz meʒųrd bį hɋf đɛ ɋrk betwen đem. Đat iz tɯ sa, it iz hɋf az lɋrj az đɛ aŋgl mad bį tω radiį tɯ đɛ endz ov đɛ kər .

8. Hwen đɛ kərd iz ɑ dįameter, đɛ aŋgl iz meʒųrd bį hɋf ov hɋf đɛ sȩrkl; đat iz, đɛ aŋgl iz ɑ rįt aŋgl. Ɑ tanjent at đɛ end ov ɑ dįameter must əlwaz bɛ at rįt aŋglz tɯ đɛ dįameter. And yω wil ezili sɛ đat it must bɛ sω wiđ ɑ radius. Ɑ tanjent at đɛ end ov ɑ radius iz at rįt aŋglz tɯ đɛ radius.

9. Đąr kan bɛ tanjents tɯ uđer kurvz az weḷ az tɯ sȩrklz. Ɑ strat lįn đat just tuçez ɑ kurv; ər, if đɛ

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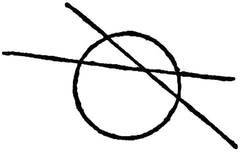
tɯ ɑ sȩrkl. ʕHʊ iz đɛ aŋgl bɛtwɛn ɑ tanjent and ɑ kərd meʒųrd. Drə on đɛ bωrd ɑ sȩrkl wiđ ɑ kərd, and ɑ tanjent at đɛ end ov it. Drə radiį tɯ đɛ endz ov đɛ kərd. Σω mɛ hwiç tω aŋglz in đat figyųr ʃɯd bɛ đɛ wun just dubl đɛ uđer. ʕHwot iz đɛ aŋgl mad bį ɑ dįameter wiđ ɑ tanjent at đɛ end ov it. ʕBį ɑ radius wiđ ɑ tanjent at đɛ end ov it. ʕHwot iz ɑ tanjent tɯ eni kurv.

kurv wj̦ndz, ɑ strat lj̦n gωiŋ ŧrω ɑ pωnt in ɑ kurv in ꝺe sam direkʃon az ꝺe kurv at ꝺat pωnt, iz kɵld ɑ tanjent tω ꝺe kurv.

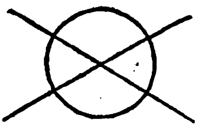
ЄAPTER XXI.

MΩR ABȢT KƟRDZ AND TANJENTS.



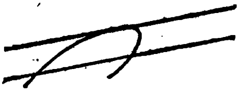
1. Hwen tω kɵrdz kros ɛç uꝺer, ꝺe aŋgl ꝺa mak iz meʒụrd bj̦ hɑf ꝺe *sum* ov ꝺe ɑrks betwen ꝺa̤r endz. Ꝺat iz, if tω strat lj̦nz kros ɛç uꝺer insj̦d ov ɑ sẹrkl, ꝺa̤r aŋgl iz meʒụrd bj̦ hɑf ꝺe *sum* ov ꝺe ɑrks betwen ꝺem; if ꝺa kros ȣtsj̦d ꝺe sẹrkl, ꝺe aŋgl iz meʒụrd bj̦ hɑf ꝺe *diferens* ov ꝺe ɑrks.

2. Hwen ꝺe kɵrdz ɑr bωŧ dj̦ameterz, ꝺe ɑrks ɑr ɛkwal, and hɑf ꝺe sum iz just wun ɑrk; sω ꝺat ꝺe aŋgl iz meʒụrd bj̦ wun ɑrk; and ꝺat, yώ nω, iz ꝺe fẹrst ŧiŋ ꝺat yω lẹrnd abȣt ꝺe meʒụr ov an aŋgl,—ꝺat an aŋgl, wiꝺ its vẹrteks in ꝺe senter, iz meʒụrd bj̦ ꝺe ɑrk betwen its sj̦dz.



3. Ɑrks ɋr pɋrts ov sẹrkumferensez. Pɋrts ov uđer kurvz ɋr keld ɋrks ov đꝏz kurvz; and strat lįnz jꝏniŋ đɛ endz ov đꝏz ɋrks ɋr keld kerdz ov đꝏz ɋrks. Hwen wɛ spek ov an ɋrk, wɛ men ɑ pes ov ɑ sẹrkl; and if wɛ wiʃ tu spek ov ɑ pes ov an elips, wɛ kel it an ɋrk ov an elips. Yꝏ wil lẹrn hwot an elips iz, ɑfter ɑ hwįl.

4. In everi ɋrk ov eni kurv đɋr must be at lest wun plas at hwiç đɛ tanjent iz paralel tu đɛ kerd ov đat ɋrk. Nɤ turn bɑk tu çapter III., sekʃon forɵ, and yꝏ wil fįnd đat hwot į hav nɤ told yꝏ iz just đɛ sam az đat, ꝏnli in diferent laŋgwaj.



5. If đɛ kerd ov eni ɋrk ov eni kurv be mꝏvd, kɛpiŋ it paralel tu its fẹrst pꝏziʃon, it wil kut of eđer ɑ loŋger er els ɑ ʃerter ɋrk. If mꝏvd wun wa it kuts of ɑ loŋger ɋrk; if mꝏvd đɛ uđer wa, it kuts of ɑ ʃerter ɋrk. If mꝏvd sꝏ az tu kut of ɑ ʃerter ɋrk, wɛ kan mꝏv it sꝏ fɋr, kɛpiŋ it stil paralel tu its fẹrst

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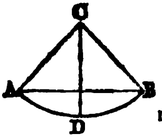
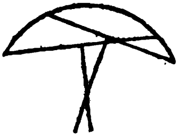
ʕ Hɤ dꝏ yꝏ mezʒɋr đɛ aŋglz ov tꝏ kerdz đat kros eç uđer. ʕ Hɤ iz it hwen đɛ kerdz ɋr dįameterz. If ɑ kerd ov an ɋrk, er eni kįnd ov kurv bɛ mꝏvd, kɛpiŋ it paralel tu its fẹrst pꝏziʃon, until it kuts of nꝏ ɋrk, ʕ hwot duz đɛ kerd bɛkum. If į gꝏ tu ɑ plas egzaktli nerɵ from mɛ, and yet at nꝏ pɋrt ov mį jurni travel nerɵ, ʕ hwot must hav bin trꝏ ov mį rꝏd. (Ɨ must hav turnd ɑ kerner in it.) If į travel ꝏver ɑ wįndiŋ rꝏd wiđɤt kernerz, and fįnd mįself at nįt in ɑ plas just est ov mį pωziʃon, ɗat it wil kut of nω ɋrk at el, but be tanjent tɯ ɗe kurv at ɗat pωnt hwąr ɗe kurv gωz in ɗe sam direkʃon az ɗe kerd. Yω kan ɛzili perform ɗis proses ov mωviŋ ɑ kerd, and kepiŋ it paralel tɯ itself, bį dreiŋ eni kurv yω çωz on yωr slat, and mωviŋ ɑ stregt ŧred akros it.

6. Let us nɤ gω bak tɯ sęrklz. If ɑ radius be dren ŧrω ɗe midl ov ɑ kerd, it wil be at rįt aŋglz tɯ ɗe kerd, and it wil end at ɗe midl ov ɗe ɋrk. Let us supωz ɗat, in ɗis figyųr, ɗe radius C D gωz ŧrω ɗe midl ov ɗe kerd A B. Ɗen, į sa ɗat ɗe aŋglz at E wil be rįt aŋglz, and ɗe ɋrk A D wil be ɛkwal tɯ ɗe ɋrk D B.



7. On ɗe uɗer hand, if we dre ɑ strat lįn ŧrω ɗe midl ov ɑ kerd, at rįt aŋglz tɯ ɗe kerd, it wil pɑs ŧrω ɗe senter ov ɗe sęrkl.

8. Ɗis givz an ɛzi wa tɯ fįnd ɗe senter ov ɑ sęrkl hwen yω hav an ɋrk ov ɗe sęrkl. Yω hav ωnli tɯ dre tω kerdz not paralel tɯ ɛç uɗer, (ɗe lɑrjer ɗe aŋgl ɗa mak ɗe beter,) and ɗen dre lįnz ŧrω ɗe midl ov ɛç kerd at rįt aŋglz tɯ



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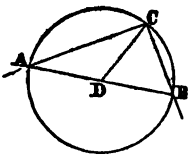
stɑrtiŋ, ? Hwot dω yω nω ov ɗe direkʃon ov mį rωd. (It must hav gon ɛst at sum pωnt ov ɗe wa.) ? Hwot kan yω tel me abɤt ɗe radius ɗat pases ŧrω ɗe midl ov ɑ kerd. ? Hwot đe kerd. Az boŧ đez lįnz pas ŧrω đe senter ov đe sęrkl, đe senter must be at đe pωnt hwąr đe lįnz kros eç uđer.

9. If yω kan get ɑ kąrd, suç az biznes men hav advertįzments printed ǫn, yω kan yųz đe loŋger sįd az ɑ rωler bį hwiç tɯ mak ɑ strat lįn; and, bį setiŋ đe ʃort sįd kąrfɯli on đis lįn, yω kan dre, bį đe loŋ sįd, ɑ lįn at rįt aŋglz tɯ yωr fęrst wun. Yω kan probabli fįnd an ωld printed kąrd bį ɑskiŋ for it, and it wil sęrv az ɑ rωler and skwąr. Đen yω kan dre ʃort ąrks bį yωr ʆ, and fįnd đe senterz bį sekʃon at.

ÇAPTER XXII.

INSKRĪBD POLIGONZ.

1. Hwen ɑ trįaŋgl haz its vęrtisez in ɑ sęrkumferens, it iz sed tɯ be inskrįbd in đe sęrkl. In uđer wurdz, hwen đe sįdz ov ɑ trįaŋgl ąr kerdz in ɑ sęrkl, đe trįaŋgl iz sed tɯ be inskrįbd in đe sęrkl. A B C iz an inskrįbd poligon.



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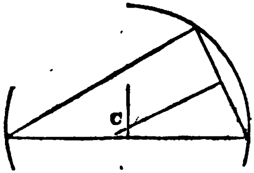
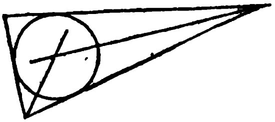
ov ɑ strat lįn at rįt aŋglz tɯ ɑ kerd ŧrω đe midl ov it. If yω fįnd an ąrk dren on đe blakbωrd, hω kan yω fįnd đe senter ov it.

2. If we wont tɯ put ɑ sẹrkl rɜnd ɑ trịaŋgl, sɷ đat đe trịaŋgl ʃal be inskrịbd in đe sẹrkl, we kan ezili dɷ it bị rememberiŋ đat đe sịdz ov đe trịaŋgl wil be kerdz in đe sẹrkl, and sɷ we kan fịnd đe senter ov đe sẹrkl, tɯ put wun leg ov ɜr kumpasez at, bị sekʃon at ov đe last çapter. We hav ɷnli tɯ dre lịnz at rịt aŋglz tɯ đe midl ov tɷ sịdz ov đe trịaŋgl, and đe pœnt hwɑ̣r đez lịnz kros, at c, wil be đe senter ov ɑ sẹrkl, hɷz sẹrkumferens wil pas ŧrɷ đe ŧre vẹrtisez ov đe trịaŋgl. Trị đis wiŧ trịaŋglz dren on đe grɜnd, and yɷ wil đen fịnd đat, bị putiŋ wun fut ov yɷr ʃiŋgl kumpas at c, yɷ kan dre ɑ sẹrkumferens ŧrɷ đe vẹrtisez.

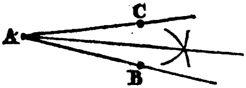


3. If, on đe uđer hand, we wiʃ tɯ inskrịb ɑ sẹrkl in đe trịaŋgl, sɷ đat eç sịd ov đe trịaŋgl ʃal be tanjent tɯ đe sẹrkl, we must dre lịnz divịdiŋ tɷ ov đe aŋglz ov đe trịaŋgl intɯ hɑ̣vz, and đe pœnt hwɑ̣r đez lịnz kros eç uđer wil be đe senter ov đe sẹrkl.



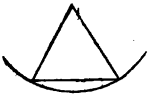
¿Hwot is ment bị an inskrịbd trịaŋgl. ¿Hwɑ̣r dɷ yɷ sa đat đe vẹrtisez ov an inskrịbd trịaŋgl ɑ̣r. ¿Hwot ɑ̣r đe sịdz. If yɷ fịnd ɑ trịaŋgl redi dren, ¿hɜ kan yɷ fịnd đe senter ov ɑ

4. But yꝏ dɷ not nɷ, perhaps, hɤ tɯ divįd an aŋgl intɯ hqvz. Supɷz, đen, đat C A B iz đɛ aŋgl yꝏ wiʃ tɯ divįd. Meʒųr A B and A C ov ɛkwal leŋŧs. Put wun fɯt ov ɑ pqr ov kumpasez at B, and wiđ đɛ uđer fɯt skraç ɑ litl qrk nɛr hwot yꝏ ŧiŋk iz đɛ midl ov đɛ aŋgl. Nɤ put wun fɯt at C, and, wiđ yꝏr kumpasez ɷpn egzaktli az wįd az bɛfɷr, mak anuđer qrk krosiŋ đɛ fęrst. Ɑ strat lįn from A ŧrꝏ đɛ pɷnts hwqr đɛ qrks kros wil divįd đɛ aŋgl intɯ tꝏ ɛkwal pqrts.



5. Đɛ lqrjest trįaŋgl đat kan bɛ inskrįbd in ɑ sęrkl iz an ɛkwilateral trįaŋgl. Sɷ đat đqr qr tꝏ kįndz ov trįaŋglz in hwiç đɛ ɛkwilateral trįaŋgl iz lqrjest; namli, đɛ įsɷperimetrikal, and đɷz inskrįbd in wun sęrkl.

6. Yꝏ remember, į hɷp, đat đɛ radius ov ɑ sęrkl iz đɛ kerd ov ɑ siksŧ pqrt ov đɛ sęrkumferens. If yꝏ drɵ an qrk loŋ ɛnuf tɯ drɵ ɑ kerd ov đɛ sam leŋŧ az ɑ radius, and đen drɵ radiį tɯ đɛ endz ov đis kerd, yꝏ wil mak an ɛkwilateral, and, đqrfɷr, an ɛkwiaŋ-



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sęrkl hꝏz sęrkumferens wil pas ŧrꝏ đɛ vęrtisez. ¿Hɤ wil yꝏ fįnd đɛ senter ov ɑ sęrkl tɯ hwiç đɛ sįdz ov đɛ trįaŋgl wil bɛ tanjent. ¿Hwot iz ment bį ɑ sęrkl inskrįbd in ɑ trįaŋgl. ¿Hɤ kan yꝏ divįd an aŋgl intɯ hqvz. Let đɛ pųpilz ilustrat az gųlɑr trįaŋgl. Ɛç aŋgl wil bɛ wun ŧẹrd ov tω rįt aŋglz; bekɵz đɛ ŧrɛ ɋr ɛkwal tɯ ɛç uđer, and đɛ ŧrɛ tɯgeđer ɋr ɛkwal tɯ tω rįt aŋglz. If ŧrɛ ov đem wɯd bɛ ɛkwal tɯ tω rįt aŋglz, siks wɯd bɛ ɛkwal tɯ fωr rįt aŋglz. And sω đɛ kɵrnerz ov siks suç trįaŋglz wɯd fil up ɵl đɛ spas abɤt đɛ senter ov ɑ sẹrkl, and đɛ siks kɵrdz wɯd just gω rɤnd đɛ sẹrkl.

7. If yω wiʃ tɯ drɵ đɛ lɋrjest trįaŋgl đat yω kan in ɑ sẹrkl, yω must ωpn yωr kumpasez just az wįd az đa wɯd bɛ tɯ drɵ đɛ sẹrkl, and đen step siks tįmz rɤnd đɛ sẹrkumferens, mɋrkiŋ đɛ pɷnts hwɋr đɛ fɛt ov đɛ kumpasez step. Đen jɷn everi uđer wun ov đɛz mɋrks bį ŧrɛ strat lįnz, and đa wil mak an inskrįbd ɛkwilateral trįaŋgl.

8. ʕDω yω understand hwot į men hwen į sa đat đɛ radius iz ɛkwal tɯ đɛ kɵrd ov siksti degrɛz.

đa resįt, bį dreiŋz on đɛ blakbɵrd. ʕHwot iz đɛ lɋrjest trįaŋgl đat kan bɛ pɯt in ɑ givn sẹrkl; đat iz, in ɑ sẹrkl đat yω fįnd ɵlredi drɵn. ʕHɤ loŋ iz đɛ kɵrd ov siksti degrɛz. ʕHwot pɋrt ov ɑ sẹrkumferens iz siksti degrɛz. ʕHɤ dω yω drɵ an ɛkwilateral trįaŋgl in ɑ sẹrkl.

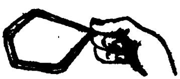
ƆAPTER XXIII.

MωR ABȣT INSKRIBD POLIGONZ.

1. Eni figyąr bȣnded bị strat lịnz iz ɑ poligon; and, if its vẹrtiseż ąr in ɑ sẹrkumferens, it iz an inskrịbd poligon.

2. Ɑ poligon ov ŧre sịdz iz keld ɑ trịaŋgl; ov fωr sịdz ɑ kwodraŋgl; ov fịv sịdz ɑ pentagon; ov siks sịdz ɑ heksagon.

3. Enị poligon ov mωr ɗan ŧre sịdz kan hav its aŋglz elterd wiɗȣt elteriŋ ɗe leŋŧ ov its sịdz. Yω remember, ị hωp, hȣ we ilustrated ɗis, hwen we wẹr studiiŋ kwodraŋglz, bị ɑ bent twig, er bị ɑ bent lamp-lịter. If ɗe twig iz bent intɯ ɗe ferm ov ɑ trịaŋgl, and its endz held tɯgeɗer, it kan not be elterd in ʃap. But, if in ɗe ferm ov eni uɗer poligon, yω kan flatn er streç it intɯ diferent ʃaps, hwiç wil not ωnli be ịsωperimetrikal, but wil hav ɗe sịdz unçanjd.

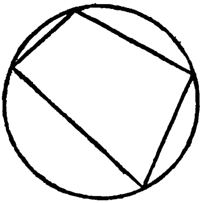


4. Hwen ɑ kwodraŋl iz pɯt intɯ ɗe ląrjest ferm it kan hav wiɗȣt elteriŋ its sịdz, it kan be inskrịbd in ɑ sẹrkl.

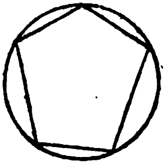
¿Hwot is ɑ poligon. ¿An inskrịbd poligon. ¿Ɑ poligon ov ŧre sịds. ¿Ov fωr sịds. ¿Ov fịv sịds. ¿Ov siks sịds. Sapes

5. Yꝏ kan pɯt ɑ sȩrkl abȣt eni trįaŋl yꝏ plɛz; but đɛ trįaŋgl iz đɛ ɷnli poligon đat kan ɵlwaz be inskrįbd in ɑ sȩrkl.

6. Hwen ɑ kwodraŋgl iz inskrįbd in ɑ sȩrkl, đɛ sum ov ɛđer tꝏ opɷzit aŋglz iz ɛkwal tɯ tꝏ rįt aŋglz. Yꝏ remember đat į hav ɵlredi tɷld yꝏ đat an aŋgl wiđ đɛ vȩrteks in ɑ sȩrkumferens iz meʒųrd bį hɋf đɛ ɋrk betwen its sįdz. But đɛ ɋrk betwen đɛ sįdz ov wun aŋgl, in đis kwodraŋgl, aded tɯ đɛ ɋrk betwen đɛ sįdz ov đɛ opɷzit aŋgl, maks up đɛ hɷl sȩrkumferens, and đɛ sum ov đɛ aŋglz iz meʒųrd bį hɋf đɛ sum ov đɛ ɋrks; đat iz, bį hɋf ɑ sȩrkumferens; đat iz, bį wun hundred and ati dɛgrɛz; đat iz, đɛ sum iz tꝏ rįt aŋglz.

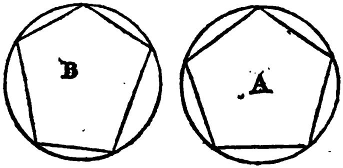


7. Eni poligon hwotever, hwen it iz inskrįbd in ɑ sȩrkl, iz in đɛ lɋrjest fɵrm đat it kan be pɯt wiđȣt ɵlteriŋ đɛ leŋŧ ov đɛ sįdz. If yꝏ had sum stiks ov diferent leŋŧs đat yꝏ wiʃt tɯ la dȣn az ɑ pla-fens upon đɛ grȣnd, yꝏ kud mak đɛ lɋrjest



đat ɑ man meʒųrs hȣ loŋ đɛ sįdz ov his fɛld ɋr, ʕwil đat tel him hȣ lɋrj đɛ fɛld is. ʕHȣ kan yꝏ ʃɷ đat it wil not, bį ɑ fɛld ov đem bį laiŋ đem in suç ɑ pωziʃon đat đɛ endz ov đɛ stiks ʃal ɵl bɛ in đɛ sęrkumferens ov wun sęrkl.

8. Hwen tω įsωperimetrikal poligonz, ov đɛ sam number ov sįdz, ɑr inskribd in sęrklz, đat poligon iz lɑrjest hwiç haz its sįdz đɛ mωst nɛrli ɛkwal. Đus, if đɛ inskrįbd pentagonz A and B węr įsoperimetrikal, A wɯd bɛ đɛ lɑrjer, bɛkɵz its sįdz ɑr ɛkwal, ɵlđω B wɯd bɛ in đɛ lɑrjer sęrkl.



9. And, đąrfωr, ov ɵl įsωperimetrikal poligonz ov đɛ sam number ov sįdz, đat wun iz đɛ lɑrjest hwiç haz its sįdz egzaktli ɛkwal, and hwiç kan bɛ inskrįbd in ɑ sęrkl.

10. Hwen ɑ poligon wiđ ɛkwal sįdz kan bɛ inskrįbd in ɑ sęrkl, it iz kɵld ɑ regyɥlɑr poligon. Not ωnli ɑr its sįdz ɛkwal tɯ ɛç uđer, but its aŋglz ɵlsω ɑr ɛkwal tɯ ɛç uđer. An ɛkwilateral trįaŋgl iz ɑ regyɥlɑr trįaŋgl, and ɑ skwąr iz ɑ regyɥlɑr kwodraŋgl.

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bent twig. ʕBut supωz hiz fɛld haz ωnli ŧrɛ sįdz. ʕSupωz it haz fωr sįdz, hwot aŋglz must đa mak tɯ hav hiz fɛld đɛ lɑrjest posibl. Hwen ɑ kwodraŋgl iz in its lɑrjest fɵrm, ʕhwot kan yω sa abȣt its vęrtisez. Supωz yω wiʃ tɯ la sevn stiks on đɛ flωr sω az tɯ enklωz đɛ mωst spas yω kan, ʕhȣ wil yω la đem. Supωz yω tak sevn uđer stiks mωr nɛrli ɛkwal in

11. Ɑ regyųlɑr poligon iz lɋrjer đan eni įsωperimetrikal poligon ov đe sam number ov sįdz. Đis iz just hwót į tωld yꝏ in đe nįnθ sekʃon.

12. Ov tꝏ įsωperimetrikal regyųlɑr poligonz, đat iz grater hwiç haz đe grater number ov sįdz. Đat iz tɯ se, ɑ skwąr iz grater đan its įsωperimetrikal ekwilateral trįaŋgl; ɑ regyųlɑr pentagon grater đan its įsωperimetrikal skwąr; and sω on.

13. Supωz yꝏ wiʃ tɯ enklωz sum land in đe midl ov ɑ grat feld, wiđ siksti panelz ov fens. If đe fens woz pɯt intɯ ɑ skwąr fɵrm, fiften panelz on ɑ sįd, it wɯd enklωz mωr đan if pɯt intɯ ɑ trįaŋgl wiđ twenti panelz on ɑ sįd; it wɯd enklωz stil mωr if pɯt rɤnd ɑ regyųlɑr pentagon, twelv panelz on ɑ sįd; stil mωr az ɑ heksagon ten panelz on ɑ sįd; and mωst ov ɵl if pɯt in ɑ regyųlɑr poligon ov siksti sįdz, wun panel on ɑ sįd.

14. We kan fansi ɑ sęrkl tɯ be ɑ regyųlɑr poligon, wiđ mωr sįdz đan kan be kɤnted, and eç sįd tꝏ ʃert tɯ be sen. Sω đat, ov ɵl įsωperimetrikal figyųrz, đe

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leŋθ, but hꝏz leŋθs aded tɯgeđer mak just az muç az đe fęrst sevn, ʕwiđ hwiç set kan yꝏ enklωz mωst spas on đe flωr. If fįv çildren tak hωld ov ɑ loŋ lꝏp ov striŋ, eç takiŋ hωld wiđ wun hand, ʕhɤ must đɑ stand sω as tɯ mak đe ωpniŋ in đe lꝏp lɋrjest. If ɑ siksθ çįld kums in and taks hωld, ʕwil đɑ mak đe lꝏp lɋrjer ɵr smeler. ʕSupωs, nɤ, đat eç çįld taks hωld wiđ bωθ handz. ʕHwot iz ment bį ɑ regyųlɑr poligon. ʕTꝏ įsωperimetrikal poligons. ʕHwot iz đe lɋrjest ov įsωperisẹrkl iz đɛ veri lɋrjest. Siksti rodz ov stωn wel kɯd not bɛ put intɯ eni ʃap and enklωz sω muç land az if it wẹr put in ɑ sẹrkl.

ĊAPTER XXIV.

ʕHɜ MUĊ FURĐER RɜND Ɑ HωP ĐAN AKROS IT.

1. Everibodi nωz đat it iz abɜt ŧrɛ tįmz az fɋr arɜnd ɑ sẹrkl az it iz akros it. But, if yω meʒųr hɜ fɋr it iz akros yωr hωp, yω wil fįnd đat ɑ striŋ ov ŧrɛ tįmz đat leŋŧ wil not kwįt gω arɜnd it.

And nɜ đɛ rest ov đis çapter wil be tω hɋrd fer çildren đat hav not lẹrnd ɑ litl Ariŧmetik. If yω hav not lẹrnd hɜ tɯ multiplį and divįd, yω wil hav tɯ gω tɯ çapter XXVI., wiđɜt understandiŋ muç abɜt đis çapter and đɛ nekst. Stil, į ŧiŋk yω wil dω wel tɯ studi đez çapterz, and lẹrn hwot yω kan ɜt ov đem, ɛvn if yω dω not nω hɜ tɯ sįfer.

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metrikal poligonz ov đɛ sam number ov sįdz. ʕHwot iz đɛ lɋrjest ov įsωperimetrikal regyųlɑr poligonz. Ov el įsωperimetrikal figyųrz ʕhwiç iz lɋrjest. Supωz yω wiʃ tɯ mak ɑ striŋ, lįiŋ on đɛ flωr, enklωz az muç spas on đɛ flωr az yω kan, ʕhɜ wil yω aranj it.

ʕHɜ muç furđer iz it rɜnd ɑ sẹrkl đan akros it. ʕHwot iz đɛ mωst komon and rufest ɑnser tɯ đis kwestyon. (3 tįmz.) ʕHwot iz ɑ mωr egzakt ɑnser. ($\frac{22}{7}$) ʕHɜ nɛrli wɯd đis

2. Đɛ sęrkumferens ov ɑ sęrkl and its dįameter ɋr nɛrli in đɛ sam prωpωrʃon az đɛ numberz 22 and 7. Sω, if yꝏ mezųr akros đɛ hꝏp, and tak ɑ striŋ ŧrɛ and wun sevnŧ tįmz đat leŋŧ, yꝏ wil fįnd đat it wil just gω rȣnd đɛ hꝏp.

3. If đɛ dįameter ov ɑ sęrkl iz sevn inçez, đɛ sęrkumferens wil lak ωnli ɑ hąrbredŧ ov twenti-tꝏ inçez; if đɛ dįameter iz sevn fɛt, đɛ sęrkumferens wil not lak đɛ bredŧ ov yꝏr slat-pensil ov beiŋ twenti-tꝏ fɛt. Nȣ ɑsk sum wun tɯ ʃω yꝏ ɑ sęrkl abȣt sevn inçəz in dįameter, suç az ɑ brekfɑst-plat; and ɑ sęrkl sevn fɛt in dįameter, suç az ɑ grat hꝏp đat ɑ man kan wek ŧrꝏ wiđ hiz hat on. Yꝏ kan đen juj hȣ nɛrli đɛ sęrkumferens and dįameter ɋr in đɛ sam prωpωrʃon az đɛ numberz twentį-tꝏ and sevn.

4. Nωbodi nωz, and nωbodi ever kan nω, egzaktli hȣ meni tįmz furđer it iz rȣnd ɑ sęrkl đan akros it. It iz veri ofn trꝏ in Jɛometri đat wɛ kan not ekspres bį figyųrz đɛ leŋŧs ov tꝏ lįnz. Đąr ɋr nω tꝏ numberz in đɛ sam prωpωrʃon az đɛ sįd and dįagωnal ov ɑ skwąr. Nωbodi kan tel egzaktli hȣ meni tįmz

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giv đɛ sęrkumferens ov ɑ sęrkl az lɋrj az ɑ plat. ʕ Hȣ nɛrli đɛ sęrkumferens ov ɑ lɋrj sistern. ʕ Hwot stil mωr egzakt ɑnser kan yꝏ giv. [3.1416.] ʕ Kan đɛ ɑnser bɛ egzaktli givn in figyųrz. ʕ Hwot Grɛk leter iz yųzd tɯ ekspres đɛ egzakt prωpωrʃon ov ɑ sęrkumferens tɯ ɑ dįameter.

loŋger đɛ dįagωnal iz đan đɛ sįd ov đɛ skwąr. And in lįk maner đąr ąr nω tꝏ numberz in đɛ sam prωpωrʃon tɯ ɛç uđer az đɛ sȩrkumferens and dįamɛter ov ɑ sȩrkl.

5. But wɛ veri ofn wont tɯ spɛk ov đis prωpωrʃon, and wɛ wont tɯ hav sum ʃørt nam før it. Jɛomɛterz hav jenerali agrɛd tɯ køl it *pį*, hwiç iz đɛ nam ov đɛ Grɛk leter før *p*, and it iz ritn π.

6. Ɨ hav tωld yꝏ đat π iz nɛrli twenti-tꝏ sevnɵs. And, in ølmωst everi kwestyon abȣt sȩrklz đat yꝏ wil wont tɯ ɑnser, đis iz egzakt ɛnuf. It iz nɛrli ɛnuf egzakt før wurkmen tɯ wurk bį in makiŋ tind wąr, ør eniɵiŋ in hwiç đɛ dįamɛter iz les đan ląrj woʃ-tubz.

7. Sω, if yꝏ nω hwot đɛ dįamɛter ov ɑ sȩrkl iz, and wont tɯ fįnd ȣt hȣ loŋ đɛ sȩrkumferens iz, yꝏ must multiplį đɛ dįamɛter bį twenti-tꝏ, and divįd đɛ produkt bį sevn. If yꝏ nω hwot đɛ sȩrkumferens iz, and wont tɯ fįnd ȣt hwot đɛ dįamɛter iz, yꝏ must multiplį đɛ sȩrkumferens bį sevn, and divįd đɛ produkt bį twenti-tꝏ.

If đąr ąr pųpilz in đɛ klas hꝏ hav studid ariɵmɛtik, let đem ɑnser suç kwestyonz az đɛ fωlωiŋ, yųziŋ twenti-tꝏ sevnɵs in đɛ hed, ør 3.1416 on đɛ slat. Đɛ simplest and best wa ov rɛdiŋ desimal frakʃonz iz simpli tɯ sa "desimal wun før wun siks."

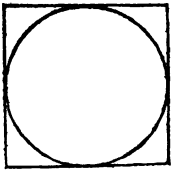
ʕ Hwot iz đɛ dįamɛter ov ɑ hꝏp 44 inçez in sȩrkumferens; ʕ Hwot iz đɛ sȩrkumferens ov ɑ hꝏp 21 inçez in dįamɛter. ʕ14 ʕ3.5 ʕ10.5, ʕ17.5, 11.

8. But, perhaps, yꝏ wil at sum tįm wiʃ tɯ be mωr egzakt, and đen it wil be beter tɯ yųz ɑ desimal frakʃon. Perhaps yꝏ hav not lẹrnd desimalz yet; but, ɑfter yꝏ hav lẹrnd đem, yꝏ ma wont tɯ yųz ɑ beter valyų ov π, and yꝏ kan đen turn bak tɯ đis paj, and fįnd đat π iz veri nɛrli ɛkwal tɯ 3.1416.

ЄAPTER XXV.

HƱ TƜ MEƷŲR ĐƐ SĮZ OV Ɑ SƐRKL.

1. Ɨ hav tωld yꝏ đat men meʒųr surfasez bį skwąrz. Đa fįnd ʊt, if đá kan, hʊ meni skwąrz it wɯd tak tɯ kuver đɛ surfasez, if đɛ sįd ov ɛç skwąr woz just wun inç, ɵr wun fɯt, ɵr wun yqrd. Ɨ hav tωld yꝏ, ɵlsω, đat wɛ kan ɛzili fįnd ʊt hʊ meni suç skwąrz it taks tɯ kuver ɑ lqrj skwąr; đat iz, wɛ kan fįnd đɛ meʒųr ov ɑ skwąr bį multiplįiŋ đɛ number ov inçez, fet ɵr yqrdz, on ɑ sįd, bį itself; đat iz, bį đɛ sam number.



2. Nʊ, if yꝏ drɵ ɑ skwąr wiđ its sįdz tanjent tɯ ɑ sẹrkl, đɛ sįdz ov đis skwąr wil bɛ ɛç ɛkwal tɯ đɛ dįameter ov đɛ sẹrkl. Đɛ meʒųr ov suç ɑ skwąr iz fʊnd,

đen, bį multiplįiŋ đɛ leŋŧ ov đɛ dįạmɛter bį itself. And đɛ meʒɥr ov đɛ sẹrkl kan bɛ fȣnd bį multiplįiŋ đɛ meʒɥr ov đɛ skwạr bį wun kwɵrter ov π.

3. Sins π iz ɑ litl mωr đan ŧrɛ, đɛ sẹrkl iz ɑ litl mωr đan ŧrɛ kwɵrterz ov đɛ skwạr. If, fɵr egzɑmpl, đɛ dįamɛter ov ɑ sẹrkl iz siks inçez, đɛ skwạr đat wil just enklωz it kontanz siks tįmz siks, ɵr ŧẹrti-siks skwạr inçez; and đɛ sẹrkl kontanz ɑ litl mωr đan ŧrɛ kwɵrterz ov đis. Ŧrɛ kwɵrterz ov ŧẹrti-siks iz twenti-sevn; and, adiŋ ɑ litl mωr, wud mak it abȣt twenti-at inçez.

4. If yω wiʃ tu bɛ mωr egzakt, yω must multiplį đɛ ŧẹrti-siks skwạr inçez bį wun kwɵrter ov twenti-tω sevnŧs; đat iz, bį ɛlevn fωrtenŧs. Ɵr, in uđer wurdz, wɛ must multiplį ŧẹrti-siks bį ɛlevn, and divįd bį fωrten, hwiç wil giv us abȣt twenti-at and wun hɑ̣f skwạr inçez fɵr đɛ sįz ov ɑ sẹrkl siks inçez in dįamɛter.

5. And, tu bɛ veri egzakt in fįndiŋ đɛ sįz ov ɑ sẹrkl, yω must multiplį đɛ dįamɛter bį itself, and đen bį đɛ desimal ·7854, hwiç iz wun kwɵrter ov 3·1416.

If đɛ klɑs hav not studid eni ariŧmɛtik, đis çapter must bɛ ωmited until ɑ rɛvɥ.

¿Hȣ dω men meʒɥr surfasez. ¿Hȣ dω yω fįnd đɛ meʒɥr ov ɑ skwạr. ¿Hȣ dω yω meʒɥr ɑ sẹrkl. Let đɛ teçer drɵ ɑ skwạr, and ɑsk, ¿Hwot figyɥr iz đis. Inskrįb ɑ sẹrkl, and ɑsk, ¿Hwot figyɥr is đis. ¿Hȣ lɑ̣rj ɑ pɑ̣rt ov đɛ skwạr duz

6. Sęrklz qr lqrjer er smeler in đe sam prωpωrʃon az đe skwąrz bilt on đąr dįameterz. And sumŧiŋ lįk đis iz trω ov el serts ov surfasez. Tω similar surfasez qr elwaz in prωpωrʃon tɯ skwąrz ov similar lįnz in đωz surfasez. If we hav tω poligonz ov đe sam ʃap, đa qr ov a sįz prωpωrʃond tɯ đe skwąrz on đąr koresponding sįdz er dįagωnalz. If a sįd in wun iz twįs az loŋ az a korespondiŋ sįd in đe uđer, đen wun poligon iz fωr tįmz đe sįz ov đe uđer, bekez twįs tω qr fωr. If wun sįd węr ŧre tįmz az loŋ az đe korespondiŋ sįd in đe uđer poligon, wun poligon wɯd be nįn tįmz az lqrj az đe uđer, bekez ŧre tįmz ŧre qr nįn.

7. Ɨ am sω aŋkʃus đat yω ʃɯd remember đis, đat į wil tel it tɯ yω agen in uđer wurdz: Θl similar surfasez qr in prωpωrʃon tɯ đe skwąrz ov korespondiŋ lįnz; sω đat we ma fįnd đe prωpωrʃon betwen đe surfas bį multiplįiŋ đe number đat ekspresez đe prωpωrʃon betwen đe lįnz bį itself.

Supωz tω dogz węr ov egzaktli đe sam ʃap, but đat wun woz twįs az hį az đe uđer. Đen its tal wɯd be twįs az loŋ az đe uđerz, its erz wɯd be twįs az loŋ, its įz twįs az wįd apqrt; and hwotever lįn yω çωz

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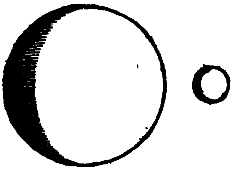
it enklωz (¾). Mωr egzaktli. ($\frac{11}{14}$) ʕStil mωr egzaktli. (·7854.) ʕBį hwot pqrt ov π must yω multiplį đe skwąr in erder tɯ fįnd đe mezʒqr ov đe sęrkl. ʕHʊ meni skwąr inçez in a sęrkl fįv inçez in dįameter. Supωz a litl man, just wun tɯ meʒɥr in wun, it wɯd bɛ twįs az loŋ az đɛ sam lįn in đɛ uđer. But its skin wɯd bɛ fωr tįmz az lqrj, đɛ surfas ov its į wɯd bɛ fωr tįmz az lqrj, it wɯd tak fωr tįmz az muç leđer tɯ mak bωts fer it, er fωr tįmz az muç lađer tɯ ʃav it; đat iz, hwotever surfas yω meʒɥrd on đɛ wun dog, yω wɯd fįnd it fωr tįmz az lqrj az đɛ sam surfas on đɛ uđer.



Supωz đat wɛ had ɑ fɯt-bel ten inçez in dįameter, and ɑ litl batiŋ-bel tω inçez in dįameter. Đɛ dįameter ov đɛ fɯt-bel, wɯd bɛ fįv tįmz az muç az đat ov đɛ batiŋ-bel, and it wɯd tak twenti-fįv tįmz az muç leđer tɯ kuver it, bekez fįv tįmz fįv iz twenti-fįv.

ÇAPTER XXVI.

ABȣT KURVATƜR.

1. Supωz đat ȣr bœ, hweliŋ hiz barω ωver đɛ lįt feln snω, went wįndiŋ abȣt đɛ feld, makiŋ ɑ kurvd trak, hwiç kurvd in sum plasez mωr đan in

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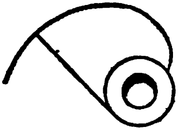
fɯt hį, and ɑ man siks fɛt hį, ʕhȣ muç mωr klođ wil it tak tɯ klođ wun đan đɛ uđer. ʕ Hȣ muç mωr yqrn tɯ nit his stokiŋz.

uɗerz. Let us supɷz ɗat hɛ began az ɗɷ hɛ wẹr gɷiŋ tɯ mak ɑ lạrj sẹrkl, but kept turniŋ ʃɵrter and ʃɵrter, and ended hwen hɛ woz turniŋ, az ɗɷ hɛ wɯd mak ɑ veri litl sẹrkl. Ɗen wɛ ʃud sa ɗat hiz trak had at fẹrst, ɑ lạrj radius ov kurvatụr, but at ɗɛ end had ɑ smɵl radius ov kurvatụr.

2. Let us supɷz ɗat ɗɛ bơ woz tịd, bị ɑ loŋ rɷp, tɯ ɗɛ truŋk ov ɑ lạrj trɛ; and ɗat, az hɛ went rɜnd and rɜnd ɗɛ trɛ, ɗɛ rɷp wɜnd up upon ɗɛ trɛ-truŋk, ʃɵrter and ʃɵrter, and drꝏ ɗɛ bơ nɛrer and nɛrer tɯ ɗɛ trɛ. Ɗen ɗɛ rɷp wɯd be ɗɛ radius ov kurvatụr ov ɗɛ bơz pạŧ.

3. Hɷld ɑ spꝏl ov ŧred stil, on yꝏr slat, and let it be ɗɛ truŋk ov ɗɛ trɛ. Ɗen tị ɗɛ end ov yꝏr slat-pensil tɯ ɗɛ end ov ɗɛ ŧred, and, bị kɛpiŋ ɗɛ ŧred tịt az yꝏ unwịnd it, yꝏ ma drɵ ɑ trak lịk ɗat ov ɗɛ bơz hwelbarɷ. Ɗɛ ŧred ɗat iz unwɜnd wil be ɗɛ radius ov kurvatụr ov ɗis mạrk. Ɗɛ radius ov kurvatụr wil be veri ʃɵrt hwạr ɗɛ pensil iz klɷs tɯ ɗɛ spꝏl, and grɷ loŋger az yꝏ unrap ɗɛ ŧred. It wil be diferent fɵr everi pơnt in ɗɛ kurv; bɛkɵz yꝏ kan not mꝏv ɗɛ pensil wiɗɜt ɛɗer wịndiŋ, ɵr els unwịndiŋ, ɗɛ ŧred.



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§ Hwot iz ɗɛ nam ov ɑ kurv ɗat bendz ɛkwali in everi pạrt.
§ Hɜ wɯd yꝏ drɵ suç ɑ kurv upon ɗɛ blakbɷrd. If ị unrap ɑ ŧred from ɑ spꝏl, hɷldiŋ ɗɛ spꝏl stil, and kɛpiŋ ɗɛ ŧred tịt,

4. Wε kel đis ŧred đε radius ov kurvatụr, bekez it iz tɯ đε kurv lịk ɑ radius tɯ đε sẹrkl. Wε kel it đe radius ov kurvatụr, bekez it ʃɷz us hʊ muɋ ɑ kurv kurvz er bendz. Hwen đe radius ov kurvatụr iz ʃert, đε kurv bendz veri muɋ; and hwen đe radius ov kurvatụr iz loŋ, đe kurv bendz les; and sɷ đe radius ov kurvatụr meʒụrz đe bendiŋ er kurvatụr ov đε kurv.

5. If wε drə ɑ sẹrkl, wiđ its senter at đe pœnt hwạr đe ŧred iz just lεviŋ đe spɷl, đat iz, hwạr đε ŧred iz tanjent tɯ đε spɷl, and mak đe radius ov đe sẹrkl just εkwal tɯ đε ŧred đat haz bin unwʊnd, đat iz, εkwal tɯ đε radius ov kurvatụr, đen đat sẹrkl wil egzaktli fit đε kurv at đe pœnt hwiɋ đe slat-pensil iz đen mạrkiŋ. Sɷ đat đε radius ov kurvatụr, at eni pœnt ov ɑ kurv, iz đε radius ov đe sẹrkl đat wil egzaktli fit đε kurv at đat pœnt.

6. Everi kurv kan be imajind az mad in ɑ similɑr wa, bị unrapiŋ ɑ striŋ of from sum uđer kurv; and đis uđer kurv iz keld đe evɷlụt ov đε fẹrst kurv.

7. But đε evɷlụt ov ɑ sẹrkl iz ɑ pœnt; bekez đe striŋ đat maks ɑ sẹrkumferens must neđer wịnd up ner unwịnd.

¶hwot sert ov ɑ kurv ʃal ị drə. ¶Hwot rεlaʃon wil đε sẹrkumferens ov đε spɷl hav tɯ đis kurv. ¶Hwot ʃal wε kel đe strat pạrt ov đε ŧred hwiɋ runz bεtwεn mị hand and đε spɷl. ¶Hwot duz đe radius ov kurvatụr meʒụr. ¶Tɯ hwot sẹrkl

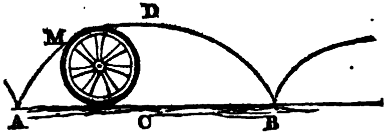
8. Đe evɷlụt ov đe bʊz trak iz đe sẹrkumferens ov đe truŋk ov đe tre; and đe evɷlụt ov đe pensil-mąrk iz đe sẹrkumferens ov đe spɷl.

9. Yɷ ma drịv a rɷ ov pinz intɯ a soft pịn bɷrd, makiŋ đe rɷ kurvd. Đen tị wun end ov a ŧred tɯ đe fɯt ov đe last pin, and đe uđer end ov đe ŧred tɯ a led-pensil ner its pɷnt. Bị kepiŋ đe striŋ stręt, and swepiŋ it rʊnd sɷ az tɯ rap up and unrap upon đe fens ov pinz, yɷ ma drɵ a kurv hɷz evɷlụt wil be đe rɷ ov pinz. Đis pensil-mąrk, yɷ wil esili se, iz mad ov litl ąrks ov sẹrklz, hɷz senterz ąr đe pinz, and đe leŋŧ ov ŧred from a pin tɯ its litl ąrk iz đe radius ov kurvatụr at đat plas.

ĊAPTER XXVII.

ABʊT A HWEL RɷLIŊ.

1. Hwen a wagon iz gɷiŋ upon a strat and level rɷd, luk at đe hed ov a spịk in đe tịr ov wun ov đe



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iz đe radius ov kurvatụr a radius. ʕHʊ dɷ we imajin el ķurvz drɵn. ʕHwot iz đe evɷlụt ov a sẹrkl. Let đe teęer prɷvịd đe bɷrd and pinz tɯ ʃɷ đe ilustraʃon ov sekʃon nịn.

hwelz, and yꝏ wil sẹ đat it mꝏvz iŋ bųtiful kurvz, makiŋ ą rω ov ąrçez đat iz kẹld a sįklɵd.

2. Đat iz tu sa, a sįklɵd iz đɛ pąŧ ov a pɵnt in đɛ sẹrkumferens ov a sẹrkl rωliŋ on a strat lįn. Yꝏ kan dro pąrt ov a sįklɵd bį putiŋ đɛ pɵnt ov yꝏr pensil intu a litl noç in đɛ ej ov a spꝏl, and tįiŋ it fast, sω đat đɛ pɵnt ov đɛ pensil ʃal be kept just at đɛ ej ov đɛ spꝏl; and đen rωliŋ đɛ spꝏl kąrfuli and slωli agenst đɛ insįd ov đɛ fram ov đɛ slat.

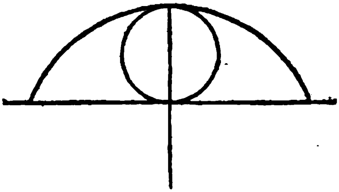
3. Yꝏ wil sẹ, į ŧiŋk, đat ɛç ąrç in đɛ sįklɵd must be just az hį from a tu D az đɛ dįameter ov đɛ sẹrkl đat maks it; and just az wįd at đɛ botom, from A tu B, az đɛ hωl sẹrkumferens ov đɛ sẹrkl.

4. But yꝏ wil hav tu studi Jɛometri a gųd hwįl, befωr yꝏ kan prꝏv đɛ uđer interestiŋ ŧiŋz hwiç į am gωiŋ tu tel yꝏ. Yꝏ kan ɛzili understand hwot į am gωiŋ tu tel yꝏ; but yꝏ kan not understand hɤ į nω it, az yꝏ kan hwot į tωld yꝏ in đɛ last sekʃon.

5. Đɛ leŋŧ ov đɛ kurv, A D B, in ɛç ąrç ov a sįklɵd, iz just fωr tįmz đɛ hįt ov đɛ ąrç; đat iz, fωr tįmz đɛ dįameter ov đɛ sẹrkl đat mad đɛ sįklɵd.

6. Đɛ hωl spas đat iz enklωzd betwen đɛ ąrç ov đɛ

Let đɛ teçer tak a tin kup, a ribon-blok, or sumŧiŋ ov đe kįnd, and rωl it kąrfuli aloŋ đɛ botom ov đɛ blakbωrd, woçiŋ and mąrkiŋ wiđ çok đɛ pąŧ ov a spot on đɛ sįd. Đen ask, ʕhwot iz đɛ nam ov đis kurv. ʕHwot iz đɛ hįt ov đɛ ąrç kompąrd wiđ đɛ *kup*, wiđ hwiç į drꝏ it. ʕHwot iz đɛ bredŧ ov đɛ ąrç at đɛ botom. ʕHwot iz đɛ leŋŧ ov đɛ kurv ov đɛ



sįklɵd and đe strat lįn on hwiç it standz, iz just ŧre tįmz az lqrj az đe sẹrkl đat mad đe sįklɵd. Sω, hwen ɑ sẹrkl iz inskrįbd betwen đe qrç and đe lįn, đe kųrius ŧre kernerd figyųrz on eç sįd ov đe sẹrkl qr eç egzaktli az lqrj az đe sẹrkl itself.

7. Nȣ, if yω hav studid Ariŧmetik, yω wil understand đat, if ɑ hwel iz ŧre fet in dįameter, đe hed ov ɑ spįk in đe tįr travelz just twelv fet from hwąr it levz đe grȣnd until it tuçez đe grȣnd agen. Đe spots hwąr it tuçez đe ẹrŧ wil be nįn fet and ŧre sevnŧs ov ɑ fut apqrt. And đe spas betwen its pqŧ and đe grȣnd wil be ŧre tįmz elevn fωrtenŧs ov nįn skwąr fet; đat iz, twenti-wun skwąr fet and ŧre fωrtenŧs ov ɑ skwąr fut.

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qrç. ʕ Hwot iz đe spas inklωzd betwen đe qrç and đe botom ov đe bωrd, (stil kompqriŋ wiđ đe kup.) Let đe teçer inskrįb ɑ sẹrkl, ov đe sįz ov đe kup, and ɑsk, ʕ Ar đez herns lqrjer er smeler đan đe sẹrkl. Supωz đat yωr hωp haz ɑ spot on wun sįd ov it, ʕ in hwot kurv wil đe spot mωv hwen đe hωp iz roliŋ strat forward. ʕ Hȣ hį wil it gω from đe grȣnd. (Djameter ov hωp.) ʕ Hȣ fqr apqrt wil đe plasez be hwąr it kumz tω đe grȣnd. ʕ Hȣ fqr wil đe spot travel in gωiŋ from wun plas tω đe nekst. (Fωr tįmz dįameter ov hωp.)

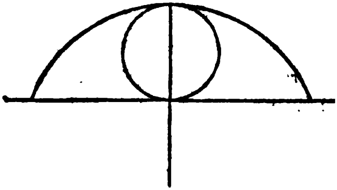
ȻAPTER XXVIII.

MꝎR ABꝊT Ɑ RꝎLIŊ HWƐL.

1. Đe hed ov ɑ spįk in đe tįr ov ɑ rꝏliŋ hwel iz mꝏviŋ, at eç instant, at rįt aŋglz tɯ ɑ lįn jꝏniŋ it tɯ đe botom ov đe hwel.

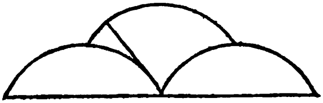
2. Đat iz tɯ sá; if ɑ strat lįn iz dren from đe botom ov đe rꝏliŋ hwel tɯ đe hed ov đe spįk, and if ɑ tanjent tɯ đe sįklꝏd iz dren frꝏ đe hed ov đe spįk, đis strat lįn wil be at rįt aŋglz tų đis tanjent.

3. And đis strat lįn iz egzaktli hɋf ov đe radius ov kurvatųr ov đat pꝏnt in đe sįklꝏd. Sꝏ đat, at đe top ov an ɋrç ov đe sįklꝏd đe radius ov kurvatųr wil be twįs đe djameter ov đe sęrkl, and as yꝏ gꝏ dꝊn đe ɋrç đe radius ov kurvatųr wil be ʃorter and ʃorter, until just at đe fɯt ov đe ɋrç đe radius ov kurvatųr wil be ov nꝏ leŋð at el.

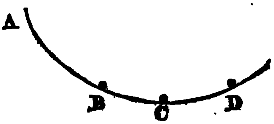


ʕHwiç ov yꝏ kan tel me hwot ɑ sįklꝏd is. If yꝏ dre ɑ lįn at rįt aŋglz tɯ ɑ sįklꝏd, ʕhwɋr wil it pas. (Frꝏ đe pꝏnt hwɋr đe sęrkl makiŋ đe sįklꝏd tuçt đe lįn on hwiç it rꝏld

4. Ðɛ evɷlɥt ov ɑ sįklɵd iz ɑ sįklɵd ov egzaktli ðɛ sam sįz. Ðat iz tɯ sa, if wɛ ʃɯd fasn ɑ striŋ in ðɛ pɵnt betwen tɷ qrçez ov ɑ sįklɵd, just loŋ ɛnuf tɯ rap on ðɛ kurv up tɯ ðɛ midl ov ðɛ qrçez, its end, az it rapt and unrapt, wɯd mɷv in ɑ sįklɵd egzaktli lįk ðat tɯ hwiç it woz fasnd.



5. If ɑ sįklɵd bɛ turnd upsįd dɤn, and wɛ fansi ðɛ insįd ov it tɯ bɛ veri eksɛdiŋli sliperi, ðen ðqr qr tɷ kɥrius θiŋz abɤt it.



If į wont tɯ slįd eniθiŋ from A dɤn tɯ B, ðqr iz nɷ kurv, nɵr strat lįn, dɤn hwiç ɑ θiŋ wɯd slįd sɷ kwikli az dɤn ðɛ sįklɵd. If ɑ hil woz holɷd ɤt in ðat ʃap, sledz wɯd run dɤn it fɑster ðan ðɑ kɯd dɤn eni uðer ʃapt hil ov ðɛ sam hįt and ðɛ sam bredθ at ðɛ botom.

6. Ðɛ sekond kɥrius θiŋ abɤt slįdiŋ on ðɛ insįd ov ɑ sįklɵd iz, ðat it taks ɵlwaz egzaktli ðɛ sam tįm tɯ slįd tɯ ðɛ botom, hɤever hį up er lɷ dɤn yɷ stqrt.

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hwen makiŋ ðat plas in ðɛ sįklɵd.) Ðɛ radius ov kurvatɥr is at rįt aŋglz tɯ ɑ kurv—¿hwot pqrt ov ðɛ radius ov ɑ sįklɵd is kut of bį ɑ strat lįn jɵniŋ ðɛ fɛt ov ðɛ qrç. (Wun hqf.) ¿Hɤ loŋ is ðɛ radius ov ðɛ sįklɵd at ðɛ top ov ðɛ qrç. ¿Hɤ loŋ at ðɛ botom. ¿Hwot is ðɛ evɷlɥt ov ɑ sįklɵd. Eksplan hwot yɷ men bį ðis. Supɷz tɷ wįrz gɵiŋ from ðɛ nerθ-ɛst kerner ov ðɛ seliŋ tɯ ðɛ sɤθ-west kerner ov ðɛ flɷr, wun wįr

If A, in đɛ last figyɋr, iz đɛ top ov suç ɑ hil, and C đɛ lɷest pɤnt, it wil tak ɑ sled egzaktli az loŋ tɯ gɷ from B tɯ C, az tɯ gɷ from A tɯ C. But đis, yɷ must rɛmember, iz ɷnli hwen wɛ imajin đɛ hil and đɛ runerz ov đɛ sledz tɯ bɛ, bɷϑ ov đem, pẹrfektli sliperi; sɷ đat đɋr ʃal bɛ nɷ rubiŋ. In đat kas, if đɛ rɷd from A tɯ C woz tɷ mįlz loŋ, it wɯd ɷnli tak ɑ sled twenti-at sekondz tɯ kum dɤn đɛ hɷl leŋϑ. And, if it stɋrts from eni uđer plas on đɛ rɷd, sa from B, it wil stil tak twenti-at sekondz tɯ get tɯ C.

7. If đɛ rɷd from A tɯ C iz hɋf ɑ mįl loŋ, ɑ sled wil kum dɤn in fɷrtɛn sekondz.

8. If ɑ bɷrd iz sed ɤt in đɛ fɵrm ov ɑ sįklɵd, and ɑ litl guter mad on đɛ insįd ov đɛ kurv, yɷ kan trį đis bį hɷldiŋ tɷ mɋrblz, sa wun at A and đɛ uđer at D, and letiŋ gɷ ov đem at đɛ sam instant. Đa wil mɛt egzaktli at C, wun kumiŋ đɛ hɷl wa A C, hwįl đɛ uđer iz kumiŋ đɛ ʃɵrt distans D C.

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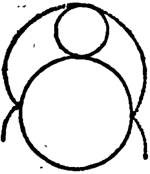
strat, đɛ uđer ɑ part ov ɑ sįklɵd, ˁ dɤn hwiç wįr wɯd eniϑiŋ slįd đɛ mɷr kwikli. Supɷz wun wįr went from đɛ nɵrϑ-ɛst kɵrner ov đɛ sɛliŋ tɯ đɛ sɤϑ-west kɵrner ov đɛ sɛliŋ, haŋiŋ dɤn in đɛ fɵrm ov ɑ hɷl ɋrç ov ɑ sįklɵd, ˁ hɤ muç loŋger wɯd it tak eniϑiŋ tɯ slįd from đɛ sɛliŋ tɯ đɛ lɷest pɋrt ov đɛ wįr, đan it wɯd tak fɵr it tɯ slįd from ɑ plas wun fɯt from đɛ midl. (Nɷ loŋger.) Đɛ tɛçer ʃɯd endevor tɯ obtan (from đɛ prɷdenʃɯl komitɛ) đɛ bɷrd dɛskrįbd in sekʃon at.

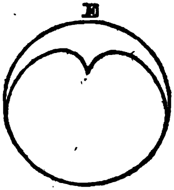
ꞒAPTER XXIX.

HWƐLZ RωLIŊ RȢND Ɑ HWƐL.



1. Hwen wun sẹrkl rωlz arȣnd anuđer, insted ov rωliŋ on ɑ strat lịn, eni pɵnt in đɛ sẹrkumferens ov đɛ rωliŋ sẹrkl travelz in ɑ kurv keld an episịklɵd. Yω kan drɵ an episịklɵd bị rωliŋ kạrfɯli đɛ spωl (wiđ ɑ pensil tịd tɯ it) arȣnd sum rȣnd ŧiŋ held stil on yωr slat.

2. Set ɑ lamp on ɑ tabl in wun kerner ov đɛ rωm, and, in đɛ fạrđest kerner ov đɛ rωm, on ɑ tabl ov nɛrli đɛ sam hịt, set ɑ brịt tin kup, er ɑ glɑs tumbler, nɛrli fɯl ov milk. On đɛ surfas ov đɛ milk yω wil sɛ ɑ brịt kurv ʃapt lịk đɛ iner lịn in đis figyụr. It iz an episịklɵd; suç az wɯd be mad bị ɑ sẹrkl ov wun kwerter đɛ dịamɛter ov đɛ kup rωliŋ on ɑ sẹrkl hạf đɛ dịamɛter ov đɛ kup. Yω kan mak

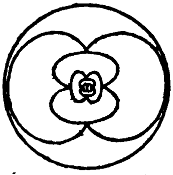


¿Hwot iz an episịkɵld. ¿Hȣ duz it difer from ɑ sịklɵd. Let đɛ tɛçer drɵ an episịklɵd az direkted in sekʃon wun, and it bį dalįt, bį setiŋ đɛ kup ov milk in đɛ sunʃįn, ęrli in đɛ merniŋ er lat in đɛ ɑfternꝏn.

3. Episįklɵdz wil bɛ ov diferent ʃaps, akerdiŋ tɯ đɛ prꝏpꝏrʃon hwiç đɛ tꝏ sęrklz bąr tɯ ɛç uđer. Đɛ smeler đɛ rꝏliŋ sęrkl iz in prꝏpꝏrʃon tɯ đɛ uđer đɛ mꝏr nɛrli wil an ąrç ov đɛ episįklɵd bɛ lįk an ąrç ov đɛ sįklɵd.

4. Đɛ evꝏlųt of an episįklɵd iz ɑ smeler episįklɵd ov đɛ sam ʃap; and đɛ evꝏlųt ov đat evꝏlųt must bɛ ɑ stil smeler episįklɵd. Sꝏ đat wɛ ma fansi episįklɵdz pakt wun wiđin anuđer lįk pil-boksez.

5. Đɛ episįklɵd ov sekʃon sekond iz sumtįmz keld bį çildren đɛ kʊz fɯt in ɑ kup ov milk. Đɛ figyųr in đɛ mąrjin reprezents đis episįklɵd wiđ its nest ov evꝏlųts pakt wun wiđin đɛ uđer. If ɑ striŋ iz fasnd at đɛ pɵnt hwąr đɛ ąrçez ov đɛ episįklɵd kum tugeđer, and iz just loŋ enuf tɯ rap rʊnd tɯ đɛ midl ov đɛ ąrç, đen, az it unraps, đɛ end wil mꝏv in ɑ ląrjer episįklɵd ov egzaktli đɛ sam ʃap.



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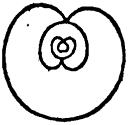
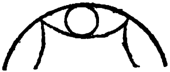
tɛç đɛ çildren tɯ dꝏ sꝏ. ʕHav eni ov yꝏ sɛn đɛ kʊz fɯt in ɑ kup ov milk. ʕHwot is đɛ jɛꝏmetrikal nam ov đis kurv. ʕHwot must bɛ đɛ prꝏpꝏrʃon betwɛn đɛ sęrklz tɯ mak đis episįklɵd. ʕHwot is đɛ evꝏlųt ov eni episįklɵd. Hwen đɛ sęrklz ąr ov đɛ sam sįz, ʕhwot wil bɛ đɛ ʃap ov đɛ episįklɵd.



6. Hwen ɗɛ sẹrklz ɑr ov ɗɛ sam sįz, ɗɛ episįklɵd wil hav but wun ɑrç. Ɗɛ endz ov ɗɛ ɑrç wil kum tɯgeɗer at ɗɛ sam pɷnt. Ɗɛ figyụr in ɗɛ mɑrjin wil ʃꝏ ɗɛ ʃap ov ɗis episįklɵd and its evꝏlụts.

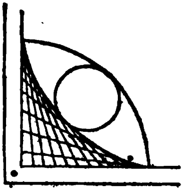
ϾAPTEŖ XXX.

OV Ɑ HWƐL RꝎLIŊ ON ƊƐ INSĮD OV Ɑ HꝎP.



1. Hwen ɑ sẹrkl rɷlz on ɗɛ insįd ov anuɗer sẹrkl, insted ov on ɗɛ ɤtsįd, ɗɛ kurv iz keld ɑ hipɷsįklɵd.

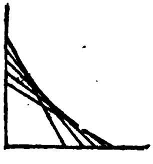
2. Supɷz yꝏr slat-pensil wẹr strat, sꝏ ɗat it wɯd lį flat on ɗɛ slat, and mak ɑ mɑrk az brɛd az ɗɛ pensil iz ꞁoŋ. Ɗen supɷz yꝏ wẹr tɯ put yꝏr pensil akros wun kɛrner ov yꝏr slat lįk ɑ hįpotɛnụs, and slįd fẹrst wun end up tɯ ɗɛ kɛrner, and ɗen ɗɛ uɗer, kɛpiŋ bꝏɵ endz el ɗɛ tįm tuçiŋ ɗɛ slat-fram. Yꝏ wɯd mak ɑ hwįt mɑrk in ɗɛ kɛrner, ov ɑ kurvd ɵrɛ-kɛrnerd ʃap, lįk ɗis figyụr. Ɗɛ kurv insįd wɯd bɛ ɑ hipɷsįklɵd.



3. If yɷ tak đe kerner ov yɷr slat fer ɑ senter, and đe leŋþ ov yɷr pensil fer ɑ radius, and dre ɑ kwerter ov ɑ sȩrkl, az ı̧ hav dun in đe lɑst figyɥr; if yɷ đen rωl on đɛ insı̧d ov đis ɑrk ɑ sȩrkl hɷz dı̧ameter iz wun hɑf đɛ leŋþ ov đɛ pensil, it wil mak đe sam hipωsı̧klɵd. Ɨ hav elsω dren đis sȩrkl in đe figyɥr.

4. Yɷ kan dre ɑ hipωsı̧klɵd bı̧ rωliŋ đɛ spωl and đe pensil on đe insı̧d ov eni litl hup held fȩrmli on đe slat. Đe rim ov đe kuver ov ɑ lɑrj wɯdn pil-boks wil mak ɑ nı̧s litl hup fer đis purpus.

5. Đe evωlɥt ov ɑ hipωsı̧klɵd iz ɑ lɑrjer hipωsı̧klɵd ov đe sam ʃap on đe ɤtsı̧d ov it; and đe hipωsı̧klɵd itself ma be fansid az đe evωlɥt ov ɑ smeler hipωsı̧klɵd wiđin it; sω đat hipωsı̧klɵdz, lı̧k episı̧klɵdz, ɑr pakt wun wiđin đe uđer, lı̧k nests ov tubz er boksez.



6. Đe hipωsı̧klɵd đat iz mad hwen đe dı̧ameter ov đe rωliŋ sȩrkl iz wun kwerter ov đe dı̧ameter ov ɑ sȩrkl đat it rωlz in, kan be mad bı̧ slı̧diŋ đe hı̧potenɥs bakwardz and fer-

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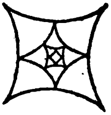
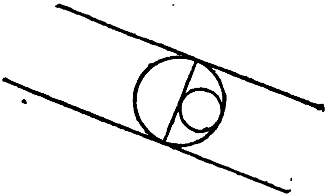
¿Hwot iz ɑ hipωsı̧klɵd. Supωz ı̧ wȩr tɯ dre ɑ hundred rı̧t trı̧-aŋglz, pɯtiŋ đɛ vȩrtisɛz ov đɛ rı̧t aŋglz tɯgeđer, wun egzaktli on anuđer, makiŋ đe hı̧potenɥsɛz ov ɛkwal leŋþ, but haviŋ nω tɷ ov đem mak đɛ sam aŋgl wiđ đɛ legz, ¿tɯ hwot kı̧nd ov ɑ hipωsı̧klɵd wɯd el đɛz hı̧potenɥsɛz be tanjent—đat wɑrdz on đe legz ov ɑ rịt trịaŋgl. Đe hịpotenụs must be kept ov đe sam leŋŧ, and it wil ɵlwaz be ɑ tanjent tɯ đe hipɷsịklɵd.



7. Đis hipɷsịklɵd ma be kɵld ɑ hipɷsịklɵd ov fɷr qrçez; bekɵz, az yɷ ma se in đe figyụr, bɷŧ it and its evɷlụts hav eç fɷr qrçez.

8. If đe dịameter ov đe spɷl iz nerli hqf đat ov đe hɯp, đe pensil wil mɷv akros đe hɯp in ɑ veri flat kurv, ɵlmɷst lịk ɑ dịameter ov đe hɯp; and đe evɷlụt at đe endz ov đe kurv wil be ɵlmɷst lịk tɷ paralel strat lịnz at rịt aŋglz tɯ đe end ov đe dịameter; sɷ đat đe striŋ unrapiŋ from đe evɷlụt wil be veri loŋ.



Hwen đe dịameter ov đe spɷl iz egzaktli hqf đat ov đe hɷp, đe hipɷsịklɵd iz ɑ strat lịn; and đe evɷlụt ov it, if yɷ kan fansi đat đạr iz eni evɷlụt, iz tɷ paralel strat lịnz at rịt aŋglz tɯ its endz.

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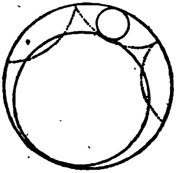
is, ʕhwot is đe prɷpɷrʃon betwen đe radịị ov đe tɷ sẹrklz. Supɷz ɑ man drez đe fɯt ov ɑ lader awa from đe sịd ov ɑ hɤs, letiŋ đe lader slip dɤn đe sịd ov đe hɤs, ʕtɯ hwot kurv in đe ạr wil đe lader be ɵl đe tịm ɑ tanjent. If đe dịameter ov đe rɷliŋ sẹrkl iz wun fifŧ đat ov đe uđer sẹrkl, ʕhɤ meni qrçez wil đe hipɷsịklɵd hav. ʕIf wun fɷrŧ. ʕIf wun ŧẹrd.

9. If đɛ dįameter ov đɛ spꝏl iz mɷr đan hɑf đat ov đɛ hɯp, it wil mak ɑ hipɷsįklɵd lįk đat mad bį ɑ smɵler spꝏl. If yꝏ hav tꝏ spꝏlz, wun ov đem az muç wįder đan đɛ radíus ov đɛ hɯp az đɛ uđer iz smɵler, sꝏ đat đɛ hɯp wil just let đɛ tꝏ spꝏlz stand in it sįd bį sįd, đen wun spꝏl wil mak egzaktli đɛ sam hipɷsįklɵd az đɛ uđer.

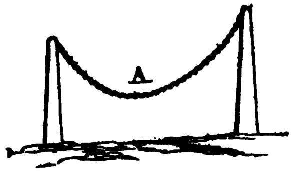
10. Đɛz tꝏ spꝏlz kan not, ov kɷrs, be bɷŧ rɷliŋ in đɛ hɯp at đɛ sam tįm; but wɛ kan ɛzili imajin tꝏ sẹrklz ov đɛ sam sįz az đɛ spꝏlz rɷliŋ in ɑ sẹrkl az lɑrj az đɛ hꝏp. Stɑrt đɛ sẹrklz from đɛ pɷziʃon in hwiç į hav drɵn đem, tu rɷliŋ in opɷzit direkʃonz, and if yꝏ rɷl đɛ litl sẹrkl faster đan đɛ lɑrj wun, sꝏ az tu mak đem get rɤnd đɛ hɯp in đɛ sam tįm, đɛ pɷnts in đɛ tꝏ sẹrklz hwiç ɑr nɤ tuçiŋ wil kɛp tugeđer ɵl đɛ tįm, makiŋ đɛ sam hipɷsįklɵd.



ʕ But hwot duz đɛ hipɷsįklɵd bɛkum hwen đɛ dįameter ov đɛ rɷliŋ sẹrkl iz wun hɑf đat ov đɛ uđer. ʕ Hɤ dꝏ đɛ evɷlɑts ov ɑ hipɷsįklɵd difer from đꝏz ov an episįklɵd. ʕ In hwot respekt ɑr đa lįk đem. ʕ Hwot must bɛ đɛ sįz ov tꝏ spꝏlz đat đa ma mak đa sam hipɷsįklɵd in đɛ sam hꝏp.

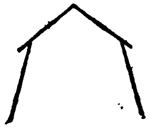
ЄAPTER XXXI.

ABʊT ɑ HAŊIŊ ЄɑN.



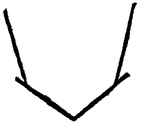
1. Hwen ɑ çan haŋz from tω pωnts not direktli under ɛç uđer, it maks ɑ bųtiful kurv kɵld ɑ katɛnari. Yω must remember đat, in ɵrder tɯ hav ɑ pęrfekt katɛnari, wɛ must tak ɑ veri fịn çan, and đen tak ωnli đɛ midl lịn in it.

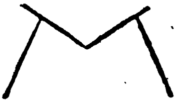
2. Supωz wɛ had fωr strat stiks jωnd tɯgeđer bị đɛ endz, sω az tɯ hav ɑ sɵrt ov çan ov fωr liŋks. Supωz đɛ midl tω węr ɛkwal in leŋð, and ɵlsω đat đɛ end wunz węr ɛkwal tɯ ɛç uđer.

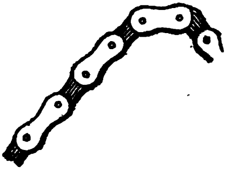


Haŋ đem bị tω pinz on ɑ level, az yω sɛ đem in đɛ figyųr, and nωtis egzaktli in hwot ʃap đɑ haŋ. Nʊ turn đem upsịd dʊn, kɛpiŋ đem in đɛ sam ʃap, az yω

ʕ Hwot iz đɛ jωmetrikal nam ov đɛ kurv mad bị ɑ haŋiŋ çan. If đɛ vęrtisɛz ov ɑ poligon węr pęrfektli limber hinjez, but đɛ sịdz stif, ʕhʊ ʃɯd wɛ plas đem tɯ mak đem stand se ðem in ðɛ nekst figyųr. Ða wil ekzaktli balans and stand lịk ðɛ rafterz ov ɑ dubl-piçt rꝏf. If yꝏ set ðɛ sịdz mꝏr nɛrli pẹrpendikyųlɑr, ðɛ top wil fel in; krɜdiŋ ðɛ sịdz apɑrt until ðɛ pꝏnt ov ðɛ top gets lꝏer ðan ðɛ top ov ðɛ sịdz, and ðen pɯliŋ ðɛ sịdz tɯgeðer agen til ða tuç at ðɛ top, and ðɛ tꝏ top pesez haŋ strat dɜn in ðɛ midl. But if, ọn ðɛ uðer hand, yꝏ len ðɛ sịdz tɯgeðer mꝏr ðan ða ʃɯd bɛ, ða wil fel tɯgeðer, krɜdiŋ ðɛ top up, until ðɛ endz ov ðɛ sịdz met, and ðɛ top pesez stand strat up, er fel tɯ wun sịd tɯgeðer. Ðɛ fꝏr stiks, hinjd tɯgeðer at ðɛ endz, wil not stand, lịk an ɑrç, unles ða mak ðɛ sam aŋglz wiđ eç uðer az ða did hwen ða wẹr haŋiŋ lịk ɑ çan.







3. And if wɛ had ɑ çan mad ov ɑ grat meni ʃert, stif pesez ov wɯd er metal, hinjd tɯgeðer bị rivets, lịk ðɛ litl çan insịd ɑ woç, wɛ kɯd mak it stand up lịk an ɑrç, if wɛ kɯd pɯt it

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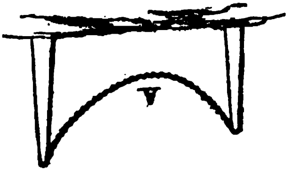
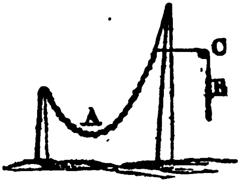
az an ɑrç. ʕDid yꝏ evẹr sɛ ɑ gambrel rꝏf. ʕDid yꝏ lịk ðɛ lɯks ov it. ʕHwot ʃap dꝏ yꝏ ŧiŋk ɑ gambrel rꝏf ʃɯd hav tɯ lɯk wel. (Ðat in hwiç ðɛ rafterz wɯd haŋ if invẹrted.)



egzaktli in đe sam form az it huŋ; đat iz, in ɑ katenari upsįd dɜn. If we ɋrç it up tꝏ step and pꝏnted, đe sįdz wil fel in; if we ɋrç it tꝏ flat, đe top wil fel in. But ɋrç it egzaktli az it huŋ, and it wil stand.



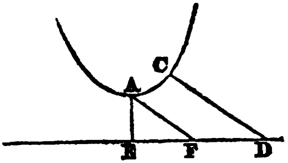
4. If we fasn wun end ov ɑ çan tu ɑ pꝏst, and haŋ đe uđer end bį ɑ ŧred from đe top ov ɑ hįer pꝏst, đe wat ov đe çan wil pul đe ŧred inward, az in đis figyųr.

But supꝏz anuđer ŧred, tįd tu đe end ov đe çan, ʃud pas ꝏver ɑ litl hwel, on ɑ level wiđ đe end ov đe çan, az at O, haviŋ ɑ pes ov đe sam kįnd ov çan huŋ tꝏ it at B. Đen yꝏ kan ɛzili se đat đe wat ov B wud pul A ɜt flater, and mak đe ŧred haŋ mꝏr nerli strat dɜn bį đe sįd ov đe pꝏst; er, if B wẹr loŋ ɛnuf, and đus hevi ɛnuf, it wud ɛvn dre đe ŧred ɜtward tꝏard O.

5. If đe pes ov çan mɋrkt B iz just loŋ ɛnuf tu pul

If į dre ɑ katenari on đe blakbꝏrd, and tel yꝏ hɜ loŋ đe radius iz at đe botom, ¿ kan yꝏ ʃꝏ me hɜ tu fįnd đe radius at đɛ end ov A egzaktli under đɛ top ov đɵ hịer pœst, sɷ az tɯ mak đɛ ŧred haŋ egzaktli strat dɜn, đen B wil bɛ just az loŋ az đɛ radius ov kurvatụr ov đɛ katɛnạri A at its lɷest pœnt.

6. Let A, in đɛ nekst figyụr, bɛ đɛ lɷest pœnt ov eni katɛnari, and C eni uđer pœnt in it yɷ plɛz.



Drɵ ɑ horizontal lịn, E D, makiŋ đɛ distans A E ɛkwal tɯ đɛ rạdius ov kurvatụr at A. Nɜ drɵ C D at rịt aŋglz tɯ đɛ katɛnari at đɛ pœnt C, and C D wil bɛ egzaktli đɛ sam leŋŧ az đɛ radius ov kurvatụr at C. Drɵ A F paralel tɯ C D, and đɛ strat lịn E F wil bɛ just az loŋ az đɛ pɵs ov çan A C.

ȻAPTER XXXII.

ƎE PAŦ OV Ɑ STƆN IN ƎE ẠR.

1. Hwen ɑ bœ tosez up hiz bel in đɛ ạr, đɛ senter ov đɛ bel mɷvz in ɑ kurv kɛld ɑ parábɑlɑ. If yɷ tos up đɛ bel on đɛ west sịd ov đɛ hɜs hwen đɛ sun

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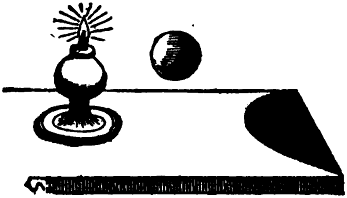
eni uđer pạrt ov đɛ çan. ʕ Kan yɷ ʃɷ mɛ hɜ tɯ fịnd ɑ strat lịn ɛkwal tɯ eni pạrt ov đɛ katɛnari.

iz setiŋ, đɛ ʃadω agenst đɛ sịd ov đɛ hȣs wil elsω mꝏv in ɑ parabωlɑ.

2. If yꝏ hωld ɑ rȣnd bel in suç ɑ poziʃon đat its ụper ej iz just az hị abuv đɛ tabl az đɛ blaz ov ɑ



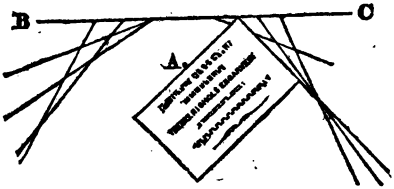
lamp iz, đen đɛ ej ov đɛ ʃadω on đɛ tabl wil bɛ ɑ parabωlɑ. Ɑ diner-plat wil elsω mak ɑ parabωlɑ in đɛ sam maner.

3. But rɛmember đat, tɯ bɛ an egzakt parabωlɑ, đɛ bel must bɛ pẹrfektli rȣnd, az nω bel kan rɛali bɛ; đɛ tabl pẹrfektli flat, az nω tabl kan rɛali bɛ; đɛ blaz ov đɛ lamp ɑ siŋgl brịt pɷnt, az nω blaz ov ɑ lamp kan bɛ. It iz ɛzi tɯ imajin egzakt figyɥrz, but đa kan never bɛ mad. Nω lịn kan bɛ dren sω fịn and trꝏ đat ɑ mịkrωskωp wɯd not fịnd ɑ bredŧ tɯ it, er waviŋ iregyɥlaritiz in it.

4. Đɛ parabωlɑ iz ɑ veri yɥsful kurv; but it wɯd bɛ difikult tɯ eksplan tɯ çildren hȣ it iz yɥzd. Ɨ ʃal tel yꝏ ov wun yɥs, befωr đɛ end ov đis buk.

ʕHwot iz đɛ jɛωmetrikal nam ov đɛ kurv in hwiç ɑ bel mꝏvz hwen tost in đɛ ạr. ʕHwiç wɯd mak ɑ mωr pẹrfekt

5. On ɑ smɷɗ bɷrd drɵ ɑ strat lịn, suç az B C.



Nɛr ɗɛ midl ov ɗɛ lịn, az at A, drịv ɑ smɵl pin. Pɯt wun ej ov ɗɛ skwąr-kɋrd agenst ɗɛ pin, and wun kɵrner on ɗɛ lịn B C, and drɵ ɑ pensil-lịn aloŋ ɗɛ ej ov ɗɛ kɋrd, beginiŋ at ɗɛ kɵrner on B C, and gɷiŋ ɵt rịt aŋglz tɯ ɗɛ ej ɗat iz agenst ɗɛ pin A. Dɷ ɗis wiɗ ɗɛ kɋrd in ɑ grat meni diferent pɷziʃonz, ɷnli kɛpiŋ ɗɛ ej agenst ɗɛ pin and ɗɛ kɵrner on B C, and yɷ wił mak ɑ plas on ɗɛ bɷrd nɛrli blak wiɗ pensil-mɋrks, wiɗ ɑ kurvd ej on ɗɛ insịd, arʊnd A, and ɗɛ kurv iz ɑ parabɷlɑ.

6. Ɗɛ pɵnt A, in ɗɛ last figyɋr, iz kɵld ɗɛ fɷkus ov ɗɛ parabɷlɑ. Ɗɛ pɵnt in ɗɛ parabɷlɑ nɛrest ɗɛ fɷkus iz kɵld ɗɛ vẹrteks ov ɗɛ parabɷlɑ. Ɗɛ lịn B C iz ɑ tanjent at ɗɛ vẹrteks.

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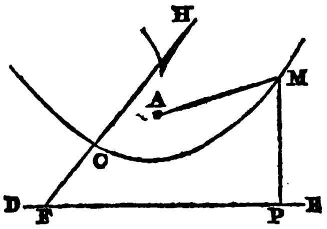
parabɷlɑ, ɑ bɵl ov led ɵr ɑ bɵl ov kɵrk. (Ov led, bɛkɵz lɛst impɛded bị ɗɛ ąr.) ʕ Hʊ must ị hɷld ɑ plat sɷ ɗat ɗɛ ej ov its ʃadɷ ʃal bɛ ɑ parabɷlɑ. ʕ Hʊ kan ị drɵ ɑ parabɷlɑ wiɗ ɑ strat ej and skwąr. ʕ Hwot iz ɗɛ vẹrteks ov ɑ parabɷlɑ. ʕ Hʊ nɛr ɗɛ vẹrteks duz ɗɛ direktriks pas. ʕ In hwot direk-



7. Let O M bɛ ɑ parabωlɑ, and let A bɛ its fωkus. Drə D E paralel tɯ đɛ tanjent at đɛ vęrteks, and az fąr from đɛ vęrteks az đɛ vęrteks iz from đɛ fωkus. Đis lįn D E iz kəld đɛ direktriks ov đɛ parabωlɑ.

8. Eni pɷnt in đɛ parabωlɑ iz just az fąr from đɛ fωkus az from đɛ direktriks. Đat iz tɯ sa, đat, if wɛ tak eni pɷnt, az M, and drə ą lįn M P at rįt aŋglz tɯ đɛ direktriks, and əlsω ɑ lįn M A, đɛ tω lįnz M P and M A wil bɛ ov egzaktli ɛkwal leŋđ.

9. Ɑ parabωlɑ ma, đąrfωr, bɛ deskrįbd az ɑ kurv, ęveri pąrt ov hwiç iz ɛkwali distant from ɑ pɷnt kəld đɛ fωkus, and from ɑ strat lįn kəld đɛ direktriks.

10. Đɛ parabωlɑ at đɛ pɷnt M maks egzaktli đɛ sam aŋgl wiđ đɛ lįn M P đat it duz wiđ đɛ lįn M A.

11. If H O iz đɛ radius ov kurvatųr at đɛ pɷnt O, and O F iz in đɛ sam strat lįn wiđ H O, đen O F iz just hąf az loŋ az H O. Đat iz tɯ sa, đat ɑ strat lįn drən

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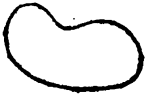
Jon duz đɛ direktriks ov ɑ parabωlɑ lį. (Paralel tɯ tanjent at đɛ vęrteks.) ʕHɤ kan yω deskrįb ɑ parabωlɑ wiđ referens tɯ its fωkus and direktriks. Let đɛ teçer kopi đɛ figyųr, and,

at rịt aŋglz tɯ eni pɵnt in ɑ parabɵlɑ, and endiŋ in ðe direktriks, iz just hąf az loŋ az ðe radius ov kurvatųr at ðat pɵnt.

ÇAPTER XXXIII.

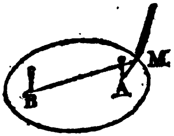
ÐE ΣADɷ OF Ɑ BƟL.

1. Eni kurv ðat runz rɜnd intɯ itself agen enklɷzez an ɷval. Ðe wurd ɷval reali menz eg-ʃapt; but, in jeometri, we yųz it fɵr eni figyųr bɜnded bị wun kurv lịn, wiðɜt eni ʃąrp kerner.



2. Ðe ʃadɷ ov ɑ rɜnd bɵl feliŋ on ɑ flat surfas, hwen ɵl ðe ʃadɷ kan be sen, iz eðer ɑ sẹrkl ɵr ɑ partikyųlar kịnd ov ɷval keld an elips. Ðe ʃadɷ ov ɑ rɜnd plat iz elsɷ an elips, hwenever ðe hɵl ʃadɷ kan be sen on wun flat surfas.

3. Yɷ kan dre an elips bị drịviŋ tɷ pinz intɯ ɑ bɵrd, az at A and B in ðe figyųr, and tịiŋ ɑ striŋ, az A M B, wun end tɯ eç pin, ðen



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dreiŋ ɑ tanjent at ðe pɵnt M, ɑsk, ¿Hɜ dus ðis tanjent divịd ðe aŋgl A M P. ¿Hɜ kan yɷ tel ðe leŋð ov ðe radius ov kurvatųr at eni pɵnt ov ɑ parabɵlɑ.

putiŋ ɑ pensil-pơnt, az at M, insịd đɛ striŋ, and streçiŋ it ʊt, and mωviŋ it rʊnd.

4. Đɛ pơnts hwạr đɛ pinz ɑ̣r plast ɑ̣r keld đɛ fωkị ov đɛ elips. Lịnz tɯ đɛ fωkị from eni pơnt in đɛ elips, az at M, mak ɛkwal aŋglz wiđ đɛ tanjent at đat pơnt.

5. Đɛ nɛrer đɛ fωkị ɑ̣r tɯgeđer, yụziŋ đɛ sam striŋ, A M B, đɛ mωr nɛrli ɑ sẹrkl duz đɛ elips bekum; sω đat if đɛ fωkị kam tɯgeđer, đɛ elips wɯd bekum ɑ sẹrkl.

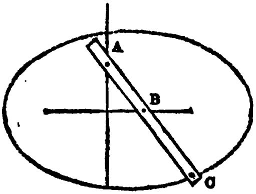
6. Đɛ furđer apɑ̣rt đɛ fωkị ɑ̣r đɛ loŋger and narωer iz đɛ elips. Hwen an elips iz veri loŋ, and veri narω in prωpωrʃon tɯ its leŋŧ, ɛç end ov đɛ elips bekumz veri muç lịk ɑ parabωlɑ.

7. Hwẹn an elips iz veri eksɛdiŋli loŋ, đɛ endz ɑ̣r sω muç lịk ɑ parabωlɑ đat ɛvn jɛometerz kel đem parabωlɑz. Wɛ kel đɛ pɑ̣ŧ ov ɑ bel tost in đɛ ɑ̣r ɑ parabωlɑ, elđω in rɛaliti it iz wun end ov ɑ veri loŋ elips, ŋerli fωr ŧʊzand mịlz loŋ, wiđ wun fωkus at đɛ senter ov đɛ ẹrŧ. But ɑ rɛal parabωlɑ iz an elips sω loŋ đat it haz nω uđer end at el; it ωnli haz wun end and wuu fωkus.

ʕHwot iz an ωval. ʕHwot iz an elips. ʕHʊ kan yω drɛ an elips wiđ ɑ striŋ and tω pinz. ʕHwot iz đɛ nam ov đɛ pơnts hwạr đɛ pinz ɑ̣r. ʕHwot aŋglz dω đɛ tω pɑ̣rts ov đɛ striŋ mak wiđ đat pɑ̣rt ov đɛ elips hwạr yωr pensil iz.

8. Ɛɛ mɷn gɷz rɜnd đɛ ẹrŧ in an elips; đɛ ẹrŧ gɷz rɜnd đɛ sun in an elips. And, if yɷ wẹr tɯ kut đɛ ẹrŧ, or sun, or mɷn, in tɷ, wiđ ɑ strat kut, đɛ kut surfas wɯd bɛ ɛđer an elips or ɑ sẹrkl, akordiŋ tɯ đɛ direkʃon in hwiç yɷ kut it. If đɛ ẹrŧ iz kut in tɷ from ɛst tɯ west, đɛ sekʃon iz ɑ sẹrkl; if in eni uđer direkʃon, an elips; for đɛ ẹrŧ iz not pẹrfektli rɜnd.

9. Kɑ̣rpenterz sumtịmz dro elipsez bị mɛnz ov ɑ bɷrd wiđ tɷ narɷ slits in it at rịt aŋglz tɯ ɛç uđer. Ɛa hav ɑ rɷler wiđ tɷ pinz in it, az at A and B, and ɑ pensil in đɛ end, az at C; and, bị mɷviŋ wun pin in wun slit, and đɛ uđer pin in đɛ uđer slit, đɛ pensil C wil mɷv in an elips.

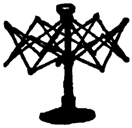


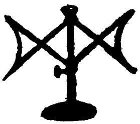
10. If yɷ kut ɑ rɜnd stik of slantiŋ wiđ ɑ ʃɑ̣rp nịf, at wun kut, đɛ kut end wil bɛ an elips.

(Ɛkwal aŋglz.) ¿Hwot uđer kurv iz đɛ end ov ɑ veri loŋ elips lịk. Hwen đɛ tɷ fɷkị ov an elips ɑ̣r brɛt nɛr tɯgeđer, ¿hwot kurv duz đɛ elips bɛkum lịk. Kan yɷ eksplaŋ hɜ ɑ kɑ̣rpenter droz an elips bị ɑ "tramel and slots." ¿Did yɷ ever nɷtis an elbɷ in ɑ stɷv-pịp. ¿Hwot iz đɛ ʃap ov đɛ sɛm arɜd đɛ elbɷ ov đɛ stɷv-pịp.

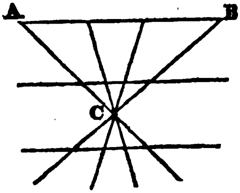
ÇAPTER XXXIV.

ĐE ƩADΩ OV Ɑ RƐL.

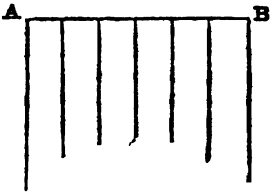




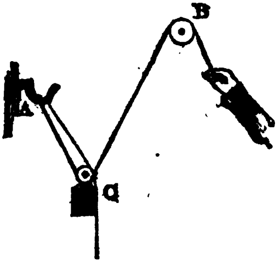
1. If ɑ rɛl fɵr wįndiŋ ŧred, suç az iz reprɛzented in đɛ figyɥr, bɛ held stedili in suç ɑ pωziʃon đat its ʃadω from ɑ lamp wil fɵl on ɑ flat wɵl, and đen set tɯ rɛvolviŋ, đɛ sįdz ov đɛ ʃadω wil bɛ kurvd, and đɛ kurv iz kɵld an hįpȩrbωlɑ.



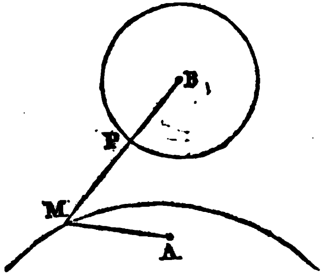
2. Supωz wɛ hav striŋz tįd, at varius plasez, on ɑ horizontal wįr, A B, and ɵl drɵn strat ŧrɯ wun pɵnt, C. Kut đem ɵl of on ɑ lįn paralel tɯ A B. Let đɛ striŋz, ɑfter bɛiŋ đus trimd, haŋ strat dʊn, and đɛ endz wil haŋ in ɑ kurv, az ʃωn in đis figyɥr, and đat kurv wil bɛ ɑ hįpȩrbωlɑ. Đis wil ɵlsω bɛ trɯ if đɛ striŋz ɥr kut of egzaktli at đɛ pɵnt C.



3. If ɑ rꝏp iz tịd tu ɑ fikst pөnt, az đe huk A, and pasez ꝏver ɑ fikst puli, az B, not on ɑ level wiđ A, đen ɑ wat on ɑ mꝏvabl puli, C, wil mꝏv, az yꝏ raz it bị puliŋ đe rꝏp ꝏver B, in ɑ hịpẹrbꝏlɑ.



4. If we dre ɑ sẹrkl rɜnd ɑ senter, B, mɋrkiŋ, elsꝏ sum pөnt ɜtsịd đe sẹrkl, az at A, and đen mak ɑ dot at everi pөnt hwiç we kan fịnd sityụated, lịk M, az

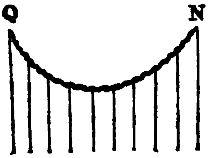


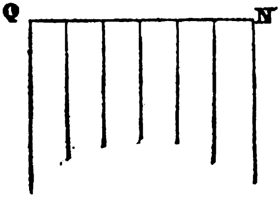
fɋr from đe pөnt A az from đe sẹrkumferens ov đe sẹrkl B, đez dots wil ol be in ɑ hịpẹrbꝏlɑ.

ʕ Did yꝏ ever se ɑ "swift" fөr wịndiŋ yɋrn. ʕ Did yꝏ ever se it standiŋ stedi on ɑ tabl, and spiniŋ rɜnd veri fast. ʕ Did yꝏ nωtis đat đe sịdz lωkt kurvd. ʕ Hwot kurv woz it. Supωz ɑ rω ov paliŋz set in ɑ strat lịn, đe midl wun đe ʃertest,

5. Yɷ sɛ, ɗen, ɗat a hịpẹrbɷla kan bɛ fansid az a parabɷla wiɗ a sẹrkumferens, insted ov a strat lịn, før a direktriks.

6. In ɗɛ last figyụr ɗɛ pɵnts A and B qr keld ɗɛ fɷkị ov ɗɛ hịpẹrbɷla. Ɗɛ kurv iz, at ȩç pɵnt, az fqr from wun fɷkus az from ɗɛ direktriks sẹrkl drɵn rɜnd ɗɛ uɗer. Ɗat iz, M P iz ov ɗɛ sam leŋŧ az M A.





7. If wɛ haŋ lịt ŧredz tɯ ȩç liŋk ov a haŋiŋ çan, suç az Q N, and kut ɗạr lɷer endz of on a level lịn, and ɗen streç ɗɛ çan ɜt pẹrfektli level, ɗɛ lɷer endz ov ɗɛ ŧred wil qrç up intɯ a hịpẹrbɷla.

8. If a rɜnd bøl haŋz egzaktli under a lamp, ɷver a tabl, its ʃadɷ on ɗɛ tabl wil bɛ a sẹrkl. But if ɗɛ bøl iz mɷvd tɯ wun sịd, ɗɛ ʃadɷ bɛkumz an elips. Nɜ raz ɗɛ bøl slɷli, and ɗɛ ʃadɷ wil begin tɯ mɷv awa from ɗɛ lamp. But wun ej wil mɷv awa muç

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and ɗɛ uɗerz just loŋ ɛnuf tɯ reç ɗɛ top ov ɗɛ midl wun, if ɗa wẹr lɛnd agenst it wiɗ ɗạr botomz standiŋ hwạr ɗa nɜ dɷ; ¶ hwot kurv wụd ɗɛ top ov suç a rɷ ov paliŋz mak. (Sekʃon tɷ.) ¶ In hwot kurv duz a mɷvabl pụli mɷv, hwen ɗɛ fikst pụli iz not on a level wiɗ ɗɛ fikst end ov ɗɛ rɷp. ¶ Hɜ faster đan đɛ uđer, sω đat đɛ elips wil grω loŋger and loŋger. And if wɛ imajin đɛ tabl tɯ bɛ sω lɑ̣rj đat wɛ kan not sɛ đɛ ejez ov it, đen, hwen đɛ uper ej ov đɛ bɵl iz just on ɑ level wiđ đɛ lamp, đɛ elips wil bɛ sω loŋ đat it wil hav nω uđer end, and đɛ end nɛrest đɛ lamp wil bɛ ɑ parábωlɑ. If wɛ raz đɛ bɵl hịer, đɛ parábωlɑ bɛkumz ɑ hịpẹ́rbωlɑ. And hwen đɛ bɵl iz razd sω hị đat its under sịd iz az hị az đɛ lamp, đɛ ʃadω wil not tuç đɛ tabl at ɵl. Đɛ parábωlɑ and hịpẹ́rbωlɑ ɑ̣r mad bị đɛ ʃadω ov đɛ lωer sịd ov đɛ bɵl.

9. If yώ tak ɑ plat insted ov ɑ bɵl, yω kan mak ɵl đɛ ʃadωz, sẹrkl, elips, parábωlɑ, and hịpẹ́rbωlɑ, bị ɑ litl panz-takiŋ, tɯ hωld đɛ plat at đɛ proper aŋgl wiđ đɛ tabl fɵr đɛ sẹrkl and elips. Fɵr đɛ parábωlɑ and hịpẹ́rbωlɑ les kɑ̣r iz rɛkwịrd; ωnli đat fɵr ɑ parábωlɑ đɛ uper ej ov đɛ plat must bɛ just az hị az đɛ lịt; and fɵr ɑ hịpẹ́rbωlɑ đɛ plat must bɛ hịer. Đɛ ʃadω ov đɛ lωer ej ov đɛ plat maks đɛ parábωlɑ ɵr hịpẹ́rbωlɑ.

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dω ɑ parábωlɑ and ɑ hịpẹ́rbωlɑ kompɑ̣r wiđ ɛç uđer. ⸮Hɤ meni fωkị haz ɑ parábωlɑ. ⸮Ɑ hịpẹ́rbωlɑ. ⸮Kan yώ tel hɤ tɯ mak ɑ hịpẹ́rbωlɑ bị ɑ çan and ŧredz. ⸮Kan yω dɛskrịb hɤ đɛ ʃadω ov ɑ bɵl, ɵr plat, ma bɛ mad tɯ grω from ɑ sẹrkl intɯ ɑ hịpẹ́rbωlɑ. ⸮Hwot uđer tώ kurvz duz it bɛkum bɛfωr bɛkumiŋ ɑ hịpẹ́rbωlɑ.

ÇAPTER XXXV.

ĐƐ KƔZ FƜT IN Ɑ KUP OV MILK.

1. Ɨ hav ɘlredi tωld yω hɤ tɯ mak đɛ brįt kurv kɘld bį çildren đɛ kɤz fɯt in ɑ kup ov milk. Ɨ hav tωld yω hɤ tɯ drɘ ɑ parabωlɑ bį drɘiŋ lįnz tanjent tɯ it until ɘl đɛ paper ɤtsįd đɛ parabωlɑ iz blaknd bį pensil mɋrks. Ɨ hav ɘlsω tωld yω hɤ tɯ drɘ ɑ hipωsįklɵd, bį pɯtiŋ ɑ ʃɘrt rωler akros wun kɘrner ov yωr slat, kɛpiŋ wun end agenst đɛ fram at đɛ botom, and đɛ uđer end agenst đɛ fram at đɛ sįd, and drɘiŋ pensil mɋrks đɛ hωl leŋŧ ov đɛ rωler on đɛ sįd nekst đɛ kɘrner. Đɛ kɘrner wil bɛkum, bį makiŋ đɛ mɋrks at ɑ grat meni diferent aŋglz wiđ đɛ fram, hwįtnd wiđ pensil mɋrks, ɘl tanjent tɯ an ɋrç ov ɑ hipωsįklɵd.

Đez ŧre kurvz ɋr, in wun respekt, alįk; đa ɋr mad bį drɘiŋ tanjents tɯ đem. Fɘr đɛ kɤz fɯt iz mad bį brįt strat lįnz ov rɛflekted lįt, ɘl tanjent tɯ an episįklɵd.

2. And hwenever lįt iz rɛflekted from đɛ insįd ov ɑ poliʃt kurv, đɛ rɛflekted lįt maks ɑ brįt kurv ov

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ʕ Hɤ ɋr kurvz drɘn bį drɘiŋ ωnli strat lįnz. ʕ Hwot iz đɛ jɛωmetrikal nam fɘr ɘl kurvz mad bį rɛflekted lįt. ʕ Hwot iz sum kįnd, just az đe lįt reflekted from đe insįd ov ɑ sęrkl maks an episįklɵd.

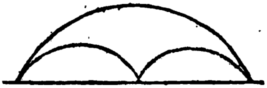
3. Kurvz mad bį reflekted lįt ąr keld kestiks. Đe kɤz fɯt in ɑ kup iz ɑ kestik mad bį ɑ sęrkl. Đe kestik mad bį ɑ sęrkl iz an episįklɵd.

4. Supωz yꝏ had ɑ tabl sω aranjd đat đe setiŋ sun ʃɯd ʃįn ωver its surfas. If on đis tabl we ʃɯd set narω strips ov tin on đąr ejez, đa wɯd reflekt đe sunlįt and mak brįt kurvz er kestiks on đe tabl.

5. If đe tin węr bent intɯ ɑ hąf ov ɑ sęrkl, đe kestik mad bį it wɯd be, az yꝏ elredi nω, an episįklɵd suç az wɯd be mad bį wun sęrkl rωliŋ on anuđęr ov twįs its dįameter.

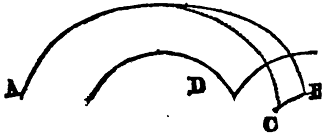
6. If anuđer strip węr bent intɯ đe ferm ov ɑ parabωlɑ, and turnd in suç ɑ direkʃon đat đe sun-lįt fel at rįt aŋglz tɯ đe direktriks ov đe parabωlɑ, đen đe kestik, insted ov beiŋ ɑ kurv, wɯd be ɑ siŋgl brįt pɵnt at đe fωkus ov đe parabωlɑ.

7. If yꝏ bend anuđer strip intɯ an ąrç ov ɑ sįklɵd, and turn đe strat lįn hwiç jɵnz đe endz ov đe ąrç at rįt aŋglz tɯ đe sunlįt, đe kestik wil be tꝏ ąrçez ov ɑ sįklɵd ov just hąf đe sįz, az ʃωn in đe figyųr.



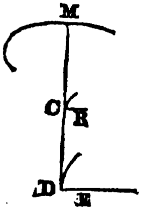
đe kestik mad bį ɑ sęrkl. ʕHwot iz đe çįldz nam fer it. ʕHɤ kɯd we aranj ɑ tabl and mak kestiks wiđ đe kurvz.

8. If đɛ sịklɵd bɛ turnd rɜnd at rịt aŋglz tɯ its last pɷziʃon, sɷ đat đɛ strat lịn jɵniŋ đɛ endz ov đɛ qrç ʃal bɛ paralel tɯ đɛ sun-lịt, đɛ kestik wil not bɛ a sịklɵd, but wil bɛ kurv konsentrik wiđ a sịklɵd ov hqf đɛ sịz. In đɛ figyɥr, A B reprɛzents đɛ strip ov tin in đɛ ferm ov a sịklɵd; C iz đɛ pɵnt ov đɛ kestik, and D iz wun qrç ov đɛ hqf-sịz sịklɵd wiđ hwiç đɛ kestik iz konsentrik.



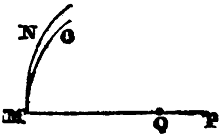
9. Hwotever kurvd ferm đɛ tin ma bɛ bent intɯ, đɛ kestik wil hav sum kɥrius propertiz, hwiç ị wil nɜ tel yɷ.

Hwąrever đɛ lịt felz at rịt aŋglz tɯ đɛ kurvd tin, az at M, đɛ kestik wil bɛ at đɛ midl ov đɛ radius ov kurvatɥr, and đɛ radius ov kurvatɥr wil bɛ tanjent tɯ đɛ kestik, az đɛ radius M D iz tanjent tɯ đɛ kestik C, just hqf wa from M tɯ D. And đɛ radius ov kurvatɥr ov đɛ kestik at đis plas, C, wil bɛ just wun-kwerter ov đɛ radius ov kurvatɥr ov đɛ evɷlɥt ov đɛ tin at M. If E iz đɛ radius ov kurvatɥr ov đɛ evɷlɥt,



ʕ Hwot iz đąr pekɥlyar abɜt dɛ kestiks ov a parabɷla. ʕ Ov a sịklɵd. ʕ Dɷ yɷ remember hwot paralel lịnz qr. ʕ Konsentrik kurvz. ʕ Hwot iz đɛ kestik ov a sịklɵd turnd endwịz đen R, đe radius ov kurvatųr ov đe kestik, wil be paralel tu E, and be wun kwerter az loŋ az R. And boŧ R and E wil be at rįt aŋglz tu đe radius M D.

But hwąrever đe sun-lįt felz az ɑ tanjent tu đe insįd ov đe kurvd strip ov tin, đe kestik wil elsω be ɑ kurv tanjent tu đe insįd ov đe tin, and its radius ov kurvatųr wil be egzaktli ŧre-kwerterz đe radius ov kurvatųr ov đe tin kurv. Ŧus, if M N iz ɑ kurv on đe insįd ov hwiç đe sunlįt felz paralel tu đe kurv at M, đen M O wil be đe kestik, and its radius, M Q, wil be egzaktli ŧre kwerterz ov M P, đe radius ov M N.



10. Ɵl ŧrω đis çapter on kestiks į hav spωkn ωnli ov đωz đat ąr mad hwen đe lįt iz at ɑ grat distans from đe poliʃt tin. If we briŋ ɑ lamp ner đe poliʃt kurv, đe kestiks mad bį đis lamp-lįt wil be veri diferent.

11. Ɑ lamp plast in đe senter ov ɑ sęrkl wud not mak ɑ kurvd kestik, but el đe lįt wud be ŧrωn bak tu wun pent, in đe senter, hwąr đe lamp itself stud.

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tu đe lįt. ¿Hwot kestik iz elwaz fermd hwen đe lįt felz perpendikyųlar tu eni pąrt ov ɑ kurv. ¿Hwen đe lįt felz paralel tu it. Let đe teçer kopi đe figyųrz, and gω over el đe pekųlyar pents ov sekʃon nįn. ¿Hwot wud be đe kestik fęrmd bį ɑ lamp in đe senter ov ɑ sęrkl. ¿In đe fωkus ov an elips. ¿In đe fωkus ov ɑ parabωlɑ. ¿Did yω ever nωtis đe reflektor in frunt ov ɑ lωkωmωtiv enjin. ¿Hwot iz it fer.

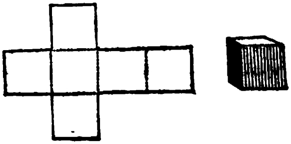
12. Ɑ lamp plast in ðɛ fωkus ov an elips wɯd not mak ɑ kurvd kɵstik, but ɵl ðɛ lįt wɯd bɛ ŧrωn tɯ wun pσnt, ðɛ uðer fωkus ov ðɛ elips.

13. Ɑ lamp plast in ðɛ fωkus ov ɑ parabωlɑ wɯd not mak ɑ kurvd kɵstik; but ɵl ðɛ lįt wɯd bɛ ŧrωn strat ɤt in paralel lįnz pęrpendikyųlɑr tɯ ðɛ direktriks ov ðɛ parabωlɑ. Fɵr ðis rɛzn men tak poliʃt rɛflektorz in ðɛ ʃap ov ɑ parabωlσd tɯ plas behįnd lamps hwen ða wont tɯ ŧrω ɤt ðɛ lįt in wun direkʃon. Ða yųz suç mirorz in lįt-hɤzez, tɯ ŧrω ðɛ lįt ɤt ωver ðɛ ωʃan; and ða yųz ðem in frunt ov lωkωmωtiv enjinz, tɯ ŧrω lįt strat fɵrwɑrd on ðɛ trak bį nįt.

ÇAPTER XXXVI.

SOLID JƐOMƐTRI.

1. Θl ðɛ figyųrz hwiç į hav tωld yω abɤt qr suç az kɯd bɛ drɵn on ɑ flat ʃɛt ov paper. Bɛfωr į finiʃ mį bɯk į wil tel yω ɑ litl abɤt ɑ fų solid bodiz.



2. Kut ɤt ov ɑ stif pɛs ov paper siks ɛkwal skwąrz, az į hav drɵn ðem hɛr; and ðen fωld ðɛ pa-

per, at ɛç lįn hwąr đɛ skwąrz jɷn ɛç uđer, tɯ ɑ rįt aŋgl. Yꝏ wil đus mak đɛ siks ɛkwal skwąrz ʃut up ɑ spas lįk ɑ boks.

3. Đis solid figyųr, bɜnded bį siks ɛkwal skwąrz, iz kɵld ɑ kųb.

4. Ɑ kųb iz takn az đɛ meʒųr ov ɵl solidz and flųidz. Ɑ galon, fɵr instans, iz tꝏ hundred and ŧȩrti-wun kųbik inçez; đat iz tɯ sa, bį ɑ galon ov wɵter wɛ mɛn wɵter ɛnuf tɯ fil tꝏ hundred and ŧȩrti-wun litl kųbz, hꝏz fasez ąr skwąr inçez. Or, fɵr anuđer instans, ɑ kɵrd ov wɯd iz wun hundred and twenti-at kųbik fɛt; đat iz, wɯd ɛnuf tɯ mak ɑ pįl az ląrj az wun hundred and twenti boksez, hꝏz sįdz ąr ɛç ɑ skwąr fɯt.

5. Yꝏ remember đat đɛ meʒųr ov ɑ skwąr iz fɜnd bį multiplįiŋ đɛ leŋŧ ov ɑ sįd bį itself. Đɛ meʒųr ov ɑ kųb iz fɜnd bį multiplįiŋ đɛ leŋŧ ov ɑ sįd twįs bį itself. If į bild ɑ kųb wiđ ɑ sįd ov ŧrɛ inçez, wun fas wil hav nįn skwąr inçez in it, and đɛ kųb wil bɛ mad up ov ŧrɛ laerz, ɛç wiđ nįn kųbik inçez in it. Sꝏ đat đɛ kųb ov ŧrɛ inçez iz ŧrɛ tįmz ŧrɛ tįmz ŧrɛ inçez; đat iz, twenti-sevn inçez. Ɑ kųb ov fꝏr inçez wɯd hav siksten skwąr inçez on ɑ sįd, and konsist

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ʕHwot iz đɛ nam ov ɑ solid bɜnded bį siks ɛkwal skwąrz. ʕHwot iz đɛ diferens betwɛn ɑ yąrd ov tap, ɑ yąrd ov ɵl-klɵŧ, and ɑ yąrd ov ȩrŧ.—Į mɛn betwɛn đɛ ŧrɛ meniŋz ov đɛ wurd yąrd in đωz frazez. ʕDꝏ yꝏ nω hɜ loŋ ɑ yąrd iz. ʕDid ov fωr laerz ov siksten kųbik inçez eç; đat iz, đe kųb ov fωr inçez iz siksti-fωr inçez.

6. Similɑr surfasez, yω remember, qr in prωpωrʃon tu đe skwąrz on korespondiŋ lįnz. Similɑr solidz qr in prωpωrʃon tu đe kųbz on korespondiŋ lįnz.

7. And az yω kan fįnd đe prωpωrʃon betwen tω similɑr surfasez bį multiplįiŋ đe number đat ekspresez đe prωpωrʃon betwen đe lįnz bį itself, sω yω kan fįnd đe prωpωrʃon betwen đe solidz bį multiplįiŋ đe number đat ekspresez đe prωpωrʃon betwen đe lįnz twįs bį itself.

8. Let us supωz, fer instans, ɑ hep ov ęrŧ siks fet hį, bilt in đe ʃap ov đe grat piramid in Ɛjipt, hwiç iz siks hundred fet hį. Đe piramid wud be wun hundred tįmz az hį az đe hep ov ęrŧ; and, beiŋ ov đe sam ʃap, wud elsω be wun hundred tįmz az wįd at đe botom. But đe piramid wud kuver ɑ hundred hundred, đat iz, ten ŧʊzand tįmz az muç land az đe hep ov ęrŧ; and it wud be ɑ hundred tįmz ten ŧʊzand, đat iz tu sa, wun milyon tįmz, az lqrj az đe hep ov ęrŧ.

9. Đe hįest mʊntenz in đe Yųnited Stats, est ov đe Misisipi river, qr abʊt ten tįmz az hį az đe grat

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yω ever se ɑ man tω yqrdz hį. Bild an ęrŧen piramid az tel az muç ɑ manz hed, and ʕhʊ muç hįer wud đe grat piramid ov Ɛjipt be. ʕHʊ muç lqrjer. ʕHwot iz đe hįest hil in đis naborhud. ʕHʊ hį iz it. ʕHʊ meni muç hilz set wun on top piramid; and if wun ov đem, wun ov đɛ Hwįt Hilz, fɵr egzampl, wẹr kut intɯ đɛ sam ʃap az đɛ grat piramid, it wɯd kuver wun hundred tįmz az muç land, and hav wun ŧʊzand tįmz az muç stωn in it, az wun ọv đɛ piramidz.

10. Đɛ hįest mʊntenz ov Tibet ɑr ṇerli fįv tįmz az hį az đɛ Hwįt Hilz ov Nų Hampʃer. If wun ov đɛ hįest mʊntenz ov Tibet wẹr ov đɛ sam ʃap az wun ov đɛ Hwįt Hilz, it wɯd hav twenti-fįv tįmz az muç land on its sįdz, and wɯd kuver twenti-fįv tįmz az muç spas. And it wɯd tak wun hundred and twenti-fįv mʊntenz lįk đɛ Hwįt Hilz tɯ mak wun ov đɛ Himalįɑ mʊntenz.

11. Ɨ wiʃ yω tɯ rɛmember veri kąrfɯli đat hwen tω ŧiŋz ɑr ov đɛ sam ʃap, ɵl korespondiŋ lįnz ɑr in đɛ sam prωpωrʃon tɯ ɛç uđer; ɵl korespondiŋ surfasez ɑr in prωpωrʃon tɯ đɛ skwąrz on đωz lįnz; and ɵl korespondiŋ solidz in prωpωrʃon tɯ đɛ kųbz on đωz lįnz.

12. Ɨ wiʃ yω ɵlsω tɯ rɛmember đat, if đɛ number ekspresiŋ đɛ prωpωrʃon betwɛn đɛ lįnz ov tω similɑr solidz bɛ multiplįd bį itself, đɛ produkt wil ekspres đɛ prωpωrʃon betwɛn đɛ korespondiŋ surfasez ; and,

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đɛ uđer wɯd it tak tɯ bɛ az hį az đɛ Hwįt Hilz. Supωz Mʊnt Woʃiŋton wẹr ov đɛ sam ʃap az ——— hil, ʕ hʊ meni suç hilz wɯd it tak tɯ bild Mʊnt Woʃiŋton. Supωz Gulįver tɯ bɛ twelv tįmz az hį az đɛ Liliputs, but ʃapt just lįk đem, if it bɛ agen multiplịd bị itself, đɛ produkt wil ekspres đɛ prɷpɷrʃon betwen đɛ korespondiŋ sɷliditiz. If đɛ dịameter ov ɑ fut-bɵl bɛ fịv tịmz đat ov ɑ batiŋ-bɵl, đɛ surfas ov đɛ fut-bɵl wil bɛ twenti-fịv tịmz az muç, and đɛ sịz ov đɛ fut-bɵl wil bɛ wun hundred and twenti-fịv tịmz az muç, az đat ov đɛ uđer bɵl.

ÇAPTER XXXVII.

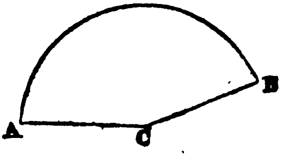
KONIK SEKƷONZ.

1. Yɷ wil veri ofn hɛr ɛr red abȣt konik sekʃonz; men began tu studi đem mɷr đan tɷ ŧȣzand yɛrz agɷ, and hav not yet lẹrnd ɵl đɛ yųsful ŧiŋz đat kan bɛ nɷn abȣt đem. Bị konik sekʃonz, wɛ men đɛ sẹrkl, đɛ elips, đɛ parabɷlɑ and đɛ hịpẹrbɷlɑ. Ɨ hav tɷld yɷ ɑ litl abȣt đɛz kurvz; ɛnuf, ị hɷp, tu mak yɷ wont tu lẹrn mɷr; and ị wil, in đis çapter, tel yɷ hwị đa qr kɵld konik sekʃonz.

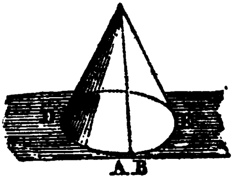
2. Kut ȣt ov pastbɷrd ɑ figyųr bȣnded bị an qrk

ʕ hȣ meni tịms lqrjer wud his nɷz bɛ đan đąrz. ʕ Hȣ meni tịmz lqrjer wud hiz ŧum-nal bɛ. ʕ Hȣ meni tịms lqrjer wud hiz fiŋger bɛ đan đąrz.

If ɑ rịt trịaŋgl kud spin rȣnd on wun leg, đɛ uđer leg wil dɛskrịb ɑ sẹrkl, and đɛ hịpotɛnųs wil gɷ rȣnd ɑ solid bodi in đɛ ąr; nȣ ʕhwot iz its jɛɷmetrikal nam. ʕ Hȣ ʃal ị and tω radiį, suç az A B C. Kurv it up ɛkwali, and jɵn đɛ ej A C tɯ đɛ ej B C. It wil đen enklωz (on ɵl sįdz but wun) ɑ spas; and đɛ figyụr đus-fɵrmd iz kɵld ɑ rįt kωn.

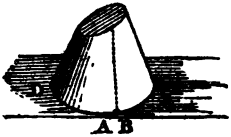


3. Set ɑ rįt kωn up upon ɑ plan, and đɛ qrk A B wil bɛkum ɑ sẹrkl, suç az D E in đɛ figyụr. If wɛ fansi ɑ pωst standiŋ strat up in đɛ senter ov ɑ sẹrkl, and ɑ loŋger strat pωl tįd bį wun end tɯ đɛ top ov đɛ pωst, hwįl đɛ uđer end just reçez đɛ sẹrkumferens ov đɛ sẹrkl; if wɛ đen fansi đis lωer end karid arɤnd đɛ sẹrkumferens, đɛ pωl wil mqrk ɤt in đɛ ąr đɛ surfas ov ɑ rįt kωn.

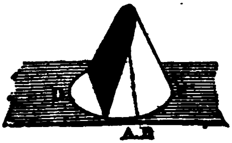


4. Kut ɑ rįt kωn in tω bį ɑ plan paralɛl tɯ đɛ plan on hwiç it sits, and đɛ kut surfas wil bɛ ɑ sẹrkl.

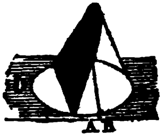
5. Kut ɑ rįt kωn in tω bį ɑ plan inklįnd tɯ đɛ plan on hwiç it standz, and đɛ kut surfas wil bɛ an elips.



“Sekʃon” mɛnz ɑ kut surfas, and “konik” mɛns beloŋiŋ tɯ ɑ kωn; sω đat yω kan nɤ understand hwį đez kurvz ąr keld "konik sekʃonz;" it iz bekez đa kan be mad bį kutiŋ ɑ kɷn.

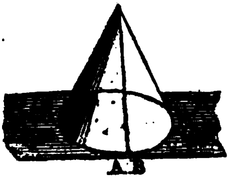


6. Kut đe kɷn bį ɑ plan paralel tu wun sįd ov đe kɷn, and đe kut surfas wil be ɑ parabɷlɑ.



7. Kut đe kɷn bį ɑ plan makiŋ ɑ smeler aŋgl wiđ đe senter-pɷst đan đe sįdz dɷ, and đe kut surfas wil be ɑ hįpęrbɷlɑ.

8. Nʊ, if yɷ wil turn bak and red çapter XXXIV., sekʃon at, agen kąrfuli, yɷ wil se đat đe ʃadɷ ov ɑ bel iz pąrt ov ɑ kɷn in đe ąr, wiđ đe vęrteks er pɷnt ov đe kɷn in đe blaz ov đe lamp, and đat đe flat tabl iz ɑ plan đat maks konik sekʃonz ov đe ʃadɷ.



9. If yɷ węr tu bend đe pastbɷrd kɷn, sɷ az tu mak D B sum uđer ʃap đan ɑ sęrkl, đe kɷn wud nɷ loŋger be ɑ rįt kɷn. Ɑ rįt kɷn haz ɑ sęrkl fer its bas, and đe vęrteks ov đe kɷn iz direktli ɷver đe senter ov đe bas.

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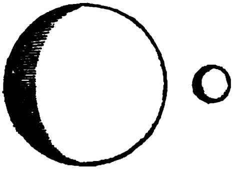
kut ɑ rįt kɷn sɷ az tu mak đe kut surfas ɑ sęrkl. ʕAn elips. ʕƑ parabɷlɑ. ʕƑ hįpęrbɷlɑ. ʕHʊ kan yɷ mak pąrt ov ɑ kɷn in đe ąr wiđ ɑ lamp and bel. ʕHʊ kan yɷ ʃɷ sekʃonz ov suç ɑ kɷn. ʕHwot iz ɑ rįt kɷn.

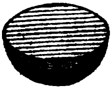
ȻAPTER XXXVIII.

ƋƐ SFƐR.

1. If ɑ sẹrkl ʃud spin rɤnd on wun ov its dịamɛterz, đɛ sẹrkumferens wɯd enklωz ɑ spas keld ɑ sfɛr.

2. Solid bodiz in đɛ ʃap ov ɑ sfɛr ɑ̣r keld belz er glωbz. Ƌɛ mɑ̣rblz wiđ hwiç bɤz pla, ɑ̣r yɥʒɥali veri pẹrfekt glωbz. Ɑ sωp-bubl blωn ɵin, and fre from eni haŋiŋ drop ov sudz, flωtiŋ in stil ɑ̣r, iz ɑ pẹrfekt sfɛr.





3. Kut ɑ sfɛr bị eni plan, and đɛ kut surfas wil be ɑ sẹrkl.

4. Hwen đɛ plan gωz dirẹktli ɵrω đɛ senter ov đɛ sfɛr, đɛ sẹrkl đus mad iz keld ɑ grat sẹrkl ov đɛ sfɛr. Ɵl grat sẹrklz in ɑ sfɛr ɑ̣r ov đɛ sam sịz az đɛ sẹrkl hwiç wɛ imajind spiniŋ tɯ kreat đɛ sfɛr.

5. Ƌɛ dịameter ov đɛ grat sẹrkl iz keld đɛ dịameter ov đɛ sfɛr, and đɛ radius ov đɛ grat sẹrkl iz keld đɛ radius ov đɛ sfɛr.

¿Hwot iz đɛ jɛωmetrikal nam fer ɑ pẹrfektli rɤnd solid. ¿Hwot dω wɛ kel ɑ solid bodi đat haz đɛ ferm ov đɛ jɛωmetrikal solid. ¿Hwot egzamplz ov belz er glωbz kan yω giv mɛ.

6. Ðɛ surfas ov ðɛ sfer iz egzaktli fωr tįmz ðɛ surfas ov ɑ grat sęrkl. Ɑ bəl þrɛ inçez in dįamɛter wɯd tak az muç leðer tɯ kuver it az wɯd mak fωr sęrklz, eç þrɛ inçez in dįamɛter; ər wun sęrkl siks inçez in dįamɛter.

7. Yω rɛmember ðat ðɛ mɛzųr ov ɑ sęrkl iz fʊnd bį multiplįiŋ ðɛ skwąr ov its dįamɛter bį wun kwərter ov π. Az ðɛ surfas ov ðɛ sfer iz fωr tįmz az grat, it iz fʊnd bį multiplįiŋ ðɛ skwąr ov its dįamɛter bį π itself. Ðɛ surfas ov ɑ bəl þrɛ inçez in dįamɛter, fər instans, wil bɛ nįn skwąr inçez multiplįd bį π.

8. Ðɛ solid mɛzųr ov ɑ sfer iz fʊnd bį multiplįiŋ ðɛ kųb ov ðɛ dįamɛter bį wun siksþ ov π. Ðɛ kųb ov ðɛ dįamɛter wil just enklωz ðɛ sfer, and eç ov ðɛ siks sįdz ov ðɛ kųb wil bɛ ɑ tanjent plan tɯ ðɛ sfer.

9. Fər rufli jujiŋ ov sęrklz and sferz wɛ kəl π abʊt þrɛ. Ðat iz tɯ sa, ɑ sęrkumferens iz ɑ litl mωr ðan þrɛ tįmz ðɛ dįamɛter; ɑ sęrkl iz ɑ litl mωr ðan þrɛ-fωrþs ov ðɛ skwąr on ðɛ dįamɛter; and ɑ sfer iz ɑ litl mωr ðan hąf ðɛ kųb on ðɛ dįamɛter.

10. Tɯ bɛ mωr egzakt, wɛ kəl π twenti-tω sevnþs. Ðat iz tɯ sa, ɑ sęrkumferens iz twenti-tω sevnþs

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¿Hʊ must ɑ sęrkl mωv tɯ hav it dɛskrįb ɑ sfer. ¿Hwot ʃap iz ðɛ sekʃon ov ɑ sfer bį ɑ plan. ¿Hwot iz ɑ grat sęrkl on ɑ sfer. ¿Hʊ ląrj iz ðɛ surfas ov ɑ sfer. ¿Hʊ ląrj iz ðɛ sωliditi ov ɑ sfer. ¿Hwot iz ðɛ ląrjest bodi ov əl haviŋ ðɛ sam surfas. ¿Intɯ hwot ʃap must į pak eniþiŋ tɯ mak it ekspωz ov đɛ dịameter; ɑ sẹrkl, ɛlevn-fɷrtɛnŧs ov đɛ skwạr on đɛ dịameter; and ɑ sfɛr, ɛlevn twenti-fẹrsts ov đɛ kɥb on đɛ dịamɛter.

11. Tɯ bɛ stil mɷr egzakt, we kɵl π, 3·1416. Đat iz tɯ sa, tɯ fịnd đɛ sẹrkumferens, multiplị ɑ dịameter bị 3·1416; tɯ fịnd đɛ sịz ov đɛ sẹrkl, multiplị đɛ dịamɛter bị itself, and đen bị ·7854; and tɯ fịnd đɛ kontents ov ɑ sfɛr, multiplị đɛ dịamɛter twịs bị itself, and đen bị ·5236.

12. Az đɛ sẹrkl iz đɛ lạrjest ov ịsɷperimetrikal figyɥrz, sɷ đɛ sfɛr iz đɛ lạrjest ov ɵl bodiz haviŋ đɛ sam amɤnt ov surfas. If yɷ rɷl ɑ pes ov puti intɯ ɑ rɤnd bɵl, it wil hav les surfas đan it kɯd hav in eni uđer fɵrm.

13. But ị ŧiŋk ị hav mad mị bɯk loŋ ɛnuf. I hɷp yɷ hav lịkt it, and ị hɷp đat at sum tịm yɷ wil studi mɷr Jɛomɛtri, and lẹrn hɤ tɯ prɷv đɛ trɷŧ ov ɵl ị hav tɷld yɷ. Yɷ wil đen fịnd đat đạr iz ɑ grat dɛl tɯ bɛ lẹrnd abɤt hwot men ɵlredi nɷ ov Jɛomɛtri, and đat đạr iz ɑ grat dɛl đat iz not nɷn, at lɛst bị eni

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lɛst surfas. If yɷ hav studid eni ariŧmɛtik, yɷ ma nɤ tel mɛ hɤ tɯ kalkɥlat rufli đɛ sẹrkumferens, đɛ surfas ov ɑ sẹrkl, đɛ surfas ov ɑ sfɛr, and đɛ sɷliditi ov ɑ sfɛr, hwen yɷ nɷ đɛ dịamɛter. ¿Hɤ ʃal wɛ kalkɥlat đɛ sam mɷr egzaktli. ¿Iz đạr eni mɷr Jɛomɛtri tɯ bɛ lẹrnd đan hwot iz tɛt in đis bɯk. ¿Ɑr đạr eni nɥ ŧiŋz yet tɯ bɛ diskuverd in Jɛomɛtri. ¿Σɯd yɷ lịk tɯ lẹrn mɷr abɤt it hwen yɷ grɷ ɷlder.

man. Ov kωrs, ðɛ grat Krɛator, hω haz mad el ŧiŋz in number, wat and meʒɥr, nωz everiŧiŋ. And ðɛ mωr wɛ nω, ðɛ mωr klerli wɛ ʃal sɛ hɤ grat iz hiz nolej, hɤ wunderful hiz wizdom, and hɤ bɥtiful ðɛ maner in hwiç hɛ haz yɥzd hwot wɛ kel Jɛometri in ðɛ fermz hɛ haz givn tɯ el ŧiŋz on ðɛ ęrŧ er in ðɛ skį.

PRAKTIKAL KWESTYONZ AND PROBLEMZ FΘR RƐVŲ BƗ ĐƐ ΩLDER SKOLɅRZ.

Đε çapters rεfǫrd tɯ ma not ɵlwaz furniʃ ɑ direkt ɑnser ɵr sωluʃon; but đa wil ɵlwaz sugjest đε trꝏ sωluʃon, hwiç is not ɵlwaz tɯ bε rεznd ʊt, but is tɯ bε sεn bȷ đε mȷndz ȷ. Similɑr kwestyonz kan bε multiplȷd indefinitli bȷ ɑ skilfɯl tεçer.

ÇAPTER III.—¿Hwǫt is đε best wa ov makiŋ ɑ gɑrden-pɑŧ strat. ¿Hʊ duz ɑ kɑrpenter mɑrk ɑ loŋ strat lȷn. ¿Hʊ wil yꝏ mak ɑ ʃɵrt strat lȷn on paper. Đε ralrωd from mȷ hʊs tɯ Boston is abʊt nȷn mȷlz loŋ, đε karij-rωd abʊt at; ¿hwiç is mωr nεrli strat. In traveliŋ đε karij rωd tɯ Boston ȷ kros đε Fiçburg ralrωd ωnli wuns. Ɨ am on đε nɵrŧ sȷd ov it at stɑrtiŋ; hwen ȷ get intɯ Boston, ¿on hwiç sȷd ov mε must ȷ lɯk fɵr đε depω. But ȷ kros đε Wɯrster ralrωd twȷs; ¿on hwiç sȷd ov mε ʃal ȷ lɯk fɵr đε depω ov đat rωd. If Boston lȷz egzaktli εst ov mȷ hʊs, ¿hʊ kan ȷ manaj tɯ drȷv mȷ hɵrs đɑr wiđʊt haviŋ his hed wuns turnd egzaktli tɯ đε εst. ¿Kan ȷ dꝏ it wiđʊt haviŋ εđer hed ɵr tal turnd tɯ đε εst.

ÇAPTER IV.—Supωs ɑ strat stik is mad tɯ turn upon ɑ pin ŧrust ŧrꝏ đε midl ov it, ¿hwiç end wil mꝏv đε faster. (Nεđer.) ¿Hwiç end wil ɵlter its direkʃon mωst rapidli. If đε pin is ŧrust intɯ ɑ strat lȷn đat wil not mꝏv, suç as ɑ krak in đɛ flɷr, ʕhwiç end ov đɛ stik wil mak đɛ lqrjer aŋgl wiđ đɛ krak.

ЄAPTER V.—If į haŋ tɷ plum-įinz from đɛ seliŋ, from nalz đat qr just wun fɯt apqrt, ʕhȣ fqr apqrt wil đɛ įinz bɛ siks fɛt belɷ đɛ seliŋ.

ЄAPTER VI.—In a trįaŋgl hɷz sįdz qr ƀrɛ, fɷr and fįv inçez, ʕhwiç iz đɛ lqrjest and hwiç đɛ smɛlest aŋgl. If a leniŋ pɷl maks an aŋgl ɛkwal tɯ wun ƀẹrd ov a rįt aŋgl, wiđ a plum-įin, ʕhwot aŋgl duz it mak wiđ a level įin pasiŋ ƀrɷ đɛ fɯt ov đɛ pɷl and under đɛ bob huŋ from its sumit. ʕHwot aŋgl wil it mak wiđ eni uđer level įin pasiŋ ƀrɷ its fɯt. ʕHwot aŋgl wil đɛ pɷl mak wiđ a level įin pasiŋ ƀrɷ its fɯt, at rįt aŋglz tɯ wun pasiŋ from đɛ fɯt under đɛ bob.

ЄAPTERZ VII., VIII., IX.—Supɷz đat į set a stak sevn fɛt hį in a level pɛs ov grȣnd, and meʒqr its ʃadɷ and đɛ ʃadɷ ov uđer ƀiŋz on level grȣnd az folɷz: Hwen đɛ ʃadɷ ov đɛ stak woz ten fɛt, đat ov đɛ hȣs woz ƀẹrti fɛt; hwen đat ov đɛ stak woz nįn fɛt, đat ov mį poplar trɛ woz fẹrti-wun; hwen đat ov đɛ stak woz at, đat ov mį çeri-trɛ woz twenti-siks; hwen đat ov đɛ stak woz siks, đat ov đɛ çurç-stɛpl woz wun hundred and twenti-wun. ʕHwot iz đɛ hįt ov đɛ stɛpl, poplar-trɛ, çeri-trɛ, and hȣs.

ЄAPTER X.—Ɨ hav wun pɷst and tɷ ralz tɯ mak a fens tɯ kɛp đɛ katl from a yuŋ trɛ đat haz spruŋ up bį đɛ fens in mį pastyqr; ʕhȣ ʃal į mak đɛ lqrjest pen fẹr it. Anuđẹr trɛ standz in đɛ midl ov đɛ fɛld; mį ɷnli matɛrialz, fẹr making a defens arȣnd it, qr ƀrɛ pɷsts, wun ral, and a pɛs ov rɷp loŋger đan đɛ ral; nȣ ʕhwot iz đɛ lqrjest trįaŋgl į kan mak.

ЄAPTER XI.—ʕHȣ lqrj iz ɛç aŋgl in an įsoselɛz rįt trįaŋgl. ʕHȣ meni suç trįaŋglz wil it tak tɯ mak a skwqr.

ʕHwot duz a kqrpenter mɛn bį a mįter-jσnt. ʕDꝏ yꝏ nω hɤ a kqrpenter maks a mįter-jσnt.

ƐAPTER XII.—If, in a fɛld wiđ fωr strat sįdz, wɛ fįnd ɵl đɛ sįdz ov đɛ sam leŋŧ, ʕhwot ma wɛ nω abɤt đɛ aŋglz. If, in suç a fɛld, wɛ fįnd đɛ sįdz ɵl ɛkwal, and tꝏ ov đɛ adjasent aŋglz ɛkwal, ʕhɤ lqrj iz ɛç aŋgl in đɛ fɛld, and hwot dꝏ yꝏ kɵl đɛ ʃap ov đɛ fɛld. Ɨ hav sɛn brasez đat wȩr ov nω yųs. ʕHwot iz đɛ proper wa tɯ mak đem. Đqr iz anuđer yųs ov ŧrɛ pσnts not egzaktli lįk đis,—ʕHwį iz a ŧrɛ-fɯted tabl ʃųr tɯ stand stedi.

ƐAPTERZ XIII. and XIV.—ʕHɤ meni yqrdz ov pantiŋ on đɛ sįd ov a hɤs fɵrti-tꝏ fɛt loŋ and twenti-wun fɛt hį. ʕHɤ meni skwąr inçez in a pan ov glas sevn bį nįn. ʕHɤ meni in a pan ov at bį ten. ʕHɤ meni fɛt ov land in a lot wiđ tꝏ sįdz ov ati fɛt ɛç and tꝏ ov ŧȩrti-nįn fɛt ɛç, if đɛ aŋglz qr suç đat đɛ ati-fɛt sįdz qr ωnli ŧȩrti-fįv fɛt apqrt. ʕHɤ dꝏ yꝏ nω đat đis lot iz a paralelωgram.

ƐAPTER XV.—ʕHɤ meni fɛt ov land in a trįaŋgl hꝏz sįdz qr ŧȩrti, fɵrti, and fifti fɛt. ʕHɤ meni in a trįaŋgl hꝏz sįdz qr twelv, fįv and ŧȩrtɛn fɛt. ʕHɤ dꝏ yꝏ nω đat đɛz trįaŋglz qr rįt trįaŋglz. Supωz đat a kwodraŋgl haz sįdz ov ŧrɛ, fωr, twelv and ŧȩrtɛn fɛt, and đat đɛ sįdz ov ŧrɛ and fωr fɛt jσn in a skwąr kɵrner, ʕhɤ meni fɛt ov land duz it inklųd. ʕHɤ dꝏ yꝏ nω đat đis kwodraŋgl kan bɛ divįded intɯ tꝏ rįt trįaŋglz.

ƐAPTERZ XVI. tɯ XXI.—If į hav a pɛs ov đɛ felωz ov a hwɛl and wont tɯ fįnd ɤt hɤ lqrj đɛ hωl hwɛl iz, ʕhwot ʃal į dꝏ. ʕHɤ wɯd yꝏ la ɤt a gqrden-pqŧ in a sȩrkl. ʕHɤ wɯd yꝏ mak tꝏ pqđz at rįt anglz tɯ ɛç uđer. ʕHɤ wil yꝏ mak a sȩrkl on đɛ blakbωrd. ʕHɤ wil yꝏ mak tꝏ pqđz run at an aŋgl ov siksti degrɛz wiđ ɛç uđer. If a stɛmbɷts tiler iz laʃt fast in eni pɷziʃon, ¿in hwot kurv wil đɛ bɷt run. ¿Hȣ wil yɷ mak đɛ sẹrkl lạrjer. If đąr is a sẹrkl dren on đɛ blakbɷrd, ¿hȣ kan ị dre a tanjent tɯ it at eni partikyɯlar spot in đɛ sẹrkumferens. Đąr ạr fɷr waz, wun from xx. 7, wun from xx. 8, wun from xxi. 5, and wun from xix. 7. Đɛs waz hav đąr speʃal advantajez and disadvantajez. Pɵnt đem ȣt.

ÇAPTERZ XXII. and XXIII.—¿Hȣ ʃal ị fịnd a pɵnt at an ɛkwal distans from ŧrɛ givn pɵnts. ¿Hȣ kan yɷ fịnd đɛ plas đat is ɛkwali distant from ŧrɛ ov đɛ kernerz ov đis rɷm. ¿Hȣ kan yɷ fịnd a spot ɛkwali distant from đɛ ɛst sịd, đɛ sȣŧ sịd, and đɛ dịagɷnal ov đɛ rɷm đat runz sȣŧ-west. ¿Hwot ŧrɛ kịnds ov poligonz *wiđ ɛkwal sịdz and ɛkwal aŋgls* kan bɛ lad tɯgeđer lịk briks in a pavment, and fil up el đɛ spas.

ÇAPTERZ XXIV. and XXV.—Supɷz đɛ ẹrŧ tɯ bɛ at ŧȣzand mịls in dịamɛter, ¿hwot iz its sẹrkumferens. ¿Hwot iz đɛ leŋŧ ov an ạrk ov sevnti degrɛz in a sẹrkl ov wun fɯt radius. ¿Hwiç waz mɷst, a skwąr ʃɛt ov tin ɛlevn inçez on a sịd, er a rȣnd pes ov đɛ sam ŧiknes ŧẹrtɛn inçez in dịamɛter. ¿If a ʃɛt ov tin fɷr inçez skwąr waz an ȣns, ¿hwot wil a ʃɛt a fɯt skwąr wa. ¿Hwot wil a sẹrkl ten inçez in dịamɛter wa. ¿Hwot is đɛ diferens betwɛn a pes ov land fɷr rods skwąr, and a pes ov fɷr skwąr rodz. ¿Hwot iz đɛ diferens bɛtwɛn a fɯt skwąr and a skwąr fɯt. If a çurç-spịr is wun hundred and twenti fɛt hị, ¿hȣ muç mɷr pant wil it tak tɯ pant đɛ çurç đan tɯ pant a model ov it, wiđ a spịr twelv inçez hị. ¿Iz a skwąr fɯt ov ʃɛt-led nesesarili in a skwąr fɛrm. ¿Hwot prɷpɷrʃon in đɛ kloŧ rɛkwịrd tɯ klɷđ a man fȷv fɛt hȷ, ɑ man fȷv fɛt ten inçez, and ɑ man siks fɛt, supωziŋ đɛ ϑrɛ men tɯ bɛ ov đɛ sam form, and drest in đɛ sam faʃon.

ÇAPTERZ XXVI., XXVII., XXVIII.—⸮Hȣ fɋr dus đɛ hed ov ɑ spȷk in đɛ tȷr ov ɑ hwɛl fωr fɛt in dȷameter travel hwȷl đɛ wagon gωz fωr mȷls on ɑ level rωd. ⸮Hwot is đɛ radius ov kurvatɋr ov its pɋϑ hwen it is at đɛ top ov đɛ hwɛl. ⸮Hwen it iz tɷ fɛt from đɛ grȣnd. ⸮Hwen it is wun fɯt from đɛ grȣnd. Ɑ pendɋlum-bob swiŋz in an ɋrk ov ɑ sęrkl; ⸮hȣ, from xxviii. 4, kan yɷ dɛvȷz ɑ plan tɯ mak it swiŋ in đɛ ɋrk ov ɑ sȷklɷd. ⸮Hwot is đɛ ʃortest pɋϑ from wun pɷnt tɯ anuđer. ⸮Is đɛ ʃortest pɋϑ elwaz kwikest. ⸮Hwen is it not, for hɷm or hwot is it not, and hwȷ not.

ÇAPTERZ XXIX. and XXX.—In ɑ maʃen kɛld ɑ fωtomɛter, ɑ bɛd is plast on đɛ rim ov ɑ hwɛl rωliŋ insȷd ov ɑ hɷp ov just dubl đɛ dȷamɛter; ⸮in hwot pɋϑ dus đɛ bɛd mɷv. In ɑ ralrωd kurv đɛ kɋrs kan not turn if đɛ radius is tɷ smol; ⸮hwot objekʃon tɯ jɷniŋ tɷ strat traks hwiç ɋr at rȷt aŋglz tɯ ɛç uđer bȷ ɑ kurvd trak mɋrkt ȣt bȷ dreiŋ meni lȷns ov ɛkwal leŋϑ akros đɛ korner.

ÇAPTER XXXI.—If ɑ rωp was wun pȣnd for ɛç yɋrd, and ȷ tȷ wun end ov it tɯ ɑ stapl in đɛ wol, ⸮hȣ hɋrd must ȷ pɯl *horizontali* in order tɯ mak đɛ radius ov kurvatɋr ten fɛt at đɛ lωest pɷnt ov đɛ rωp. ⸮Hȣ hɋrd tɯ mak đɛ radius ov kurvatɋr twenti-wun fɛt. ⸮Wun hundred and at fɛt. ⸮Hwot is đɛ radius ov kurvatɋr ov ɑ strat lȷn. ⸮Hȣ hɋrd must ȷ pɯl horizontali tɯ mak đɛ rωp strat. If ɑ çan, waiŋ tɷ pȣnds tɯ đɛ yɋrd, haŋs betwɛn tɷ pωsts ov đɛ sam hȷt, and đɛ kurvatɋr ov đɛ çan in đɛ midl has ɑ radius ov fȷv fɛt, ⸮wiđ hwot fωrs dus it drɑ in ɛç pωst. ⸮Hwot if đɛ radius is ϑęrti fɛt. ⸮If ɑ pɛs ov ϑred was at đɛ rat ov an ɤns tɯ a ŧɤzand fɛt, ʕhwot horizontal fɷrs is rɛkwɉrd tɯ mak đɛ radius ov kurvatɥr a mɉl loŋ. ʕBut hwot tɯ dro đɛ ŧred strat. In anseriŋ eni ov đez kwestyons on çapter xxxi. ʕdus it mak eni diferens hɤ loŋ, or hɤ ʃort đɛ rɷp, çan or ŧred, is.

ЄAPTERZ XXXII., XXXIII., XXXIV.—ʕHwot is đɛ pqŧ ov a rɉfl-bɛl in đɛ ąr. ʕKan it đen ever gɷ strat tɯ its mqrk. ʕKan yɷ fansi đɛ ʃap ov đɛ evɷlɥt ov an elips. If a hipɷsɉklɵd ov fɷr qrçez be yɥzd as an evɷlɥt, but đɛ striŋ is takn on tɷ opɷsit sɉdz loŋ ɛnuf tɯ rap rɤnd đɛ hɷl qrç, and on đɛ uđer sɉdz ov nɷ leŋŧ, ʕhwot sort ov a kurv wɯd it prɷdɥs. (An ɷval, but not an elips.)

ЄAPTER XXXV.—ʕHwot ʃap must a miror bɛ tɯ akt as a burniŋ miror bɉ briŋiŋ đɛ sun-lɉt tɯ a pønt. If a pes ov a holɷ sfɛr is yɥzd, at hwot distans from it wil đɛ impẹrfekt pønt ov lɉt bɛ fermd. ʕIf yɷ stand in đɛ senter ov a fɛld bɤnded bɉ a sẹrkɥlar fens, ʕhwąr wil đɛ ekɷ ov yɷr vøs sɤnd lɤdest. If đɛ wols ov a rɷm qr in đɛ ferm ov an elips, and a man stands in wun fɷkus and spɛks, ʕhwąr wil đɛ ekɷ sɤnd lɤdest. If a parabɷlɵd rɛflektor wẹr plast bɛhɉnd đɛ hwisl ov a lɷkɷmɷtiv, ʕhwot efekt wɯd it hav on đɛ sɤnd.

ЄAPTER XXXVI.—Ɑ man is sed tɯ hav borɷd a hɛp ov pɛt-mud, hwiç woz stakt in a kɥbikal ferm, fɷr fɛt on a sɉd, and tɯ hav rɛturnd tɷ hɛps, sç a kɥb ov ŧrɛ fɛt on a sɉd. ʕDid hɛ mak a just rɛturn. ʕHwot is đɛ prɷpɷrʃon betwɛn đɛ leŋŧ ov a hogshed hɷldiŋ wun hundred and twenty-fɉv galons, and a keg hɷldiŋ wun galon, if đa qr ov đɛ sam ʃap. If đɛ smelest ov đɛ ŧrɛ men menʃond on paj 129, was wun hundred and fifti pɤndz, ʕhwot dɷ đɛ uđerz wa. If a man fɉv and a hqf fɛt hɉ was wun hundred and siksti pɤndz, and ɑ mân ŧrɛ ingez tęler wɑz wun hundred and ati, ʕhwiç iz stȣ́ter in prꞷpꞷrʃon tɯ đɑ̧r hɉt.

ЄAPTER XXXVII.—Supꞷz ɑ pꞷl fasnd at đɛ end ov ɑ horizontal rɛvolviŋ qrm. If đɛ pꞷl lɉz horizontal, it kɛps in ɑ horizontal plan; if it iz vęrtikal, it deskrɉbz đɛ surfas ov ɑ vęrtikal silinder. ʕBut if it inklɉnz tꞷɑrdz đɛ senter-pꞷst abȣt hwiç đɛ qrm rɛvolvz. ʕIf it inklɉnz, but not direktli tꞷɑrd đɛ senter-pꞷst. Kut suç ɑ hɉpęrbꞷlꞷd surfas bɉ ɑ horizontal plan, and ʕhwot wil đɛ sekʃon bɛ. ʕHwot çanj, az yꝏ mꝏv đɛ vęrtikal plan tɯ and from đɛ senter ov đɛ figyqr.

ЄAPTER XXXVIII.—ʕHȣ meni kɥbik fɛt ov gas wil fil ɑ rȣnd balꝏn sevn yqrdz in dɉametɛr. ʕHȣ meni yqrdz ov silk ŧrɛ kwerterz ov ɑ yqrd wɉd wil it tak tɯ mak suç ɑ balꝏn. If đɛ ęrŧ węr at ŧȣzand mɉlz in dɉametɛr, and ɑ pęrfekt sfɛr, ʕhwot wɯd bɛ đɛ number ov skwɑ̧r mɉlz ov its surfas. ʕOv solid mɉlz in its kontents. ʕTɯ hȣ meni bɛlz ŧęrtɛn ingez in dɉametɛr wɯd it bɛ ɛkwivalent. ʕHȣ meni tunz wɯd it wa, if it węr ɛl weter, wun ŧȣzand ȣnsez tɯ ɑ kɥbik fɯt. ʕHȣ muç if ŧrɛ and wun-hqf tɉmz đat wat.

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